MAT 324, Fall 2015
PROBLEM SET 4, Due Thursday, October 8
Measurable functions
REVISED ON WED. SEPT. 30

(1) Prove that f is measurable if and only if f^3 is.

(2) Given an example of a non-measurable function g so that g^2 is measurable.

(3) If f is a continuous real-valued function on $\mathbb{R}$ prove that the set where f is differentiable is measurable (by differentiable, I mean a finite derivative).
   Hint: The definition of derivative involves a continuous parameter tending to zero. To do this problem you have to replace this with an equivalent set of conditions that only make use of countable set of parameters. This requires some care; note that the existence of the limit

$$\lim_{n \to \infty} \frac{f(x + \frac{1}{n}) - f(x)}{1/n},$$

does not imply f is differentiable at x.

(4) If f is continuous define $g(x) = f'(x)$ if f has a finite derivative at x and $g(x) = 0$ elsewhere. Prove that g is measurable. You may use the previous problem.

(5) Construct a measurable function on $\mathbb{R}$ that takes every real value at least once in every non-empty open interval.