

MAT 324, Fall 2015
PROBLEM SET 1, Due Thursday, September 3

- (1) Let $E \subset [0, 1]$ be the set of real numbers between 0 and 1 whose decimal expansion contains no 7's. Is this set open, closed or neither? Prove your answer.
- (2) Prove that the graph of a continuous function $f : \mathbb{R} \rightarrow \mathbb{R}$ is a closed set (the graph of f is the set in \mathbb{R}^2 defined as $\Gamma_f = \{(x, y) : y = f(x)\}$). Is the converse true? Prove or give a counterexample.
- (3) Let X be the set of all real roots of all polynomials with integer coefficients. Is X countable or uncountable? Explain why.
- (4) Is the function $f(x) = \sum_{n=0}^{\infty} 2^{-n} \sin(2^n x)$ Riemann integrable on $[0, 2\pi]$? Explain why or why not.
- (5) Is there a compact, uncountable set of real numbers which contains no rational numbers? Give an example or prove no such set exists.