1. Does \( \{\sin(nx)\} \) converge in the \( L^1 \) norm?

2. Give an example of a sequence of functions \( \{f_n\} \) which converges to the constant zero function in \( L^1 \), but so that \( f_n(x) \) does not converge to zero at any point of \([0, 1]\).

3. If \( f_n \to f \) in the \( L^1 \) norm, show that there is a subsequence \( f_{n_k} \) which converges a.e. to \( f \).

4. Prove that the set of continuous functions of compact support is dense in \( L^1 \).

5. For each \( 1 \leq p \leq \infty \) give an example of a function which is in \( L^p \) but not in \( L^q \) for any \( q \neq p \).

6. If \( f \in L^1 \) show that \( X_f = \{g \in L^1 : |g| \leq |f|\} \) is a compact set in the \( L^1 \) topology (i.e., a sequence in this set has a subsequence converging to a point of the set).