PROBLEM SET 5

1. Does $\{\sin(nx)\}$ converge in the L^1 norm?

2. Give an example of a sequence of functions $\{f_n\}$ which converges to the constant zero function in L^1, but so that $f_n(x)$ does not converge to zero at any point of $[0, 1]$.

3. If $f_n \to f$ in the L^1 norm, show that there is a subsequence f_{n_k} which converges a.e. to f.

4. Prove that the set of continuous functions of compact support is dense in L^1.

5. For each $1 \leq p \leq \infty$ give an example of a function which is in L^p but not in L^q for any $q \neq p$.

6. If $f \in L^1$ show that $X_f = \{g \in L^1 : |g| \leq |f|\}$ is a compact set in the L^1 topology (i.e., a sequence in this set has a subsequence converging to a point of the set).

7. If $f \in L^2$ let $T_n f$ be the nth partial sums of the Fourier series. Then $T_n f \to f$ in the L^2 norm. It is also true that $T_n f \to f$ a.e., but this is very hard to prove (one of the most famous theorems proven in the 20th century).