MIDTERM 1
MAT 142
10/07/05

Name	Sec.

ID number	TA's name

1	2	3	4	total

THIS EXAM IS WORTH 50 POINTS. PUT ALL ANSWERS IN THE SPACE PROVIDED. YOU MAY USE THE BACKS OF PAGES FOR SCRATCH WORK. NO NOTES OR CALCULATORS ARE ALLOWED.

(1) **(2 pts each, 20 pts total)** Place the letter corresponding to the correct answer in the box next to each question. Each correct answer is worth 2 points.

(i) ☐ Which of the following functions is one to one (on all real numbers)?
(a) $y = x^2$ **(b)** $y = e^{-x^2}$ **(c)** $y = \sin x$ **(d)** $y = x^2 + x^3$ **(e)** $y = x + x^3$
(f) none of these.

(ii) ☐ Which statement is false about $f(x) = (1 + \frac{1}{x})^x$, $x > 0$?
(a) f is bounded above by e **(b)** $f(x) = f(1/x)$ for all $x > 0$ **(c)** $f(1) = 2$ **(d)** f is positive **(e)** $\lim_{x\to\infty} f(x) = e$ **(f)** none of these.

(iii) ☐ If $f(x) = mx + b$ then its inverse function is
(a) $\frac{1}{m}x\frac{1}{b}$ **(b)** $\frac{1}{m}x - \frac{b}{m}$ **(c)** $\frac{1}{m}x + \frac{b}{m}$ **(d)** $\frac{b}{m}x - \frac{1}{b}$ **(e)** $\frac{b}{m}x + \frac{m}{b}$ **(f)** none of these.

(iv) ☐ Evaluate $\int_0^{\pi/6} \tan 2x dx$.
(a) $\ln \frac{1}{2}$ **(b)** $\ln 2$ **(c)** $\frac{1}{2}\ln 2$ **(d)** $\frac{\pi}{3}\ln 2$ **(e)** $2\ln 2$ **(f)** none of these.

(v) ☐ Use Simpson's rule with $n = 4$ to approximate $\int_0^2 5x^4 dx$.
(a) 30 **(b)** $30\frac{1}{2}$ **(c)** $31\frac{3}{4}$ **(d)** $31\frac{7}{8}$ **(e)** 32 **(f)** $32\frac{1}{12}$

(vi) ☐ For which of the following integrals does Simpson's rule (with any number of steps) give the the exact value?
(a) $\int_0^1 \sin(x)dx$ **(b)** $\int_0^1 e^x dx$ **(c)** $\int_0^1 x^3 dx$ **(d)** $\int_1^2 \ln x dx$ **(e)** $\int_0^1 \sqrt{x} dx$
(f) none of these.

(vii) ☐ The identity $\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$ implies that
(a) $\sinh 2x = 2\sinh x \cosh x$ **(b)** $\sinh 2x = \sinh x \cosh x$ **(c)** $\sinh 2x = \sinh^2 x \cosh^2 x$
(d) $\sinh 2x = \sinh^2 x + \cosh^2 x$ **(e)** $\sinh 2x = 2(\sinh^2 x + \cosh^2 x)$
(f) none of these.

(viii) $\square$ Find the derivative of $f(x) = (1+2x)e^{2x}$: **(a)** $2(1+2x)e^{2x}$ **(b)** $2xe^{2x}$ **(c)** $2e^{2x} + (1+2x)e^{2x}$ **(d)** $2e^{2x} + 2(1+2x)e^{2x}$ **(e)** $2e^{2x}$ **(f)** none of these.

(ix) $\square$ Find the deriviative of $f(x) = x^{\sin x}$: **(a)** $x^{\cos x}(\frac{1}{x}\cos x + \ln x \sin x)$ **(b)** $x^{\sin x}(\frac{1}{x}\sin x - \ln x \cos x)$ **(c)** $x^{\cos x}(\frac{1}{x}\sin x + \ln x \cos x)$ **(d)** $x^{\sin x}(\sin x + \frac{1}{x}\cos x)$ **(e)** $x^{\sin x}(\frac{1}{x}\sin x + \ln x \cos x)$ **(f)** none of these.

(x) $\square$ If m, n are integers then $\int_0^{2\pi} \sin(nx)\sin(mx)dx =$ **(a)** 0 **(b)** π **(c)** 0 if $n = m$ and π if $n \neq m$ **(d)** 0 of $n < m$ and π if $n \geq m$ **(e)** π if $n = m$ and 0 if $n \neq m$ **(f)** none of these.

(2) **(2 pts each, 10 pts total)** Find each of the following integrals. Put your final answer in the box.

(i) $\int \frac{1}{1+4x^2} dx$, $\boxed{}$

(ii) $\int_1^2 \frac{2}{x} \ln x dx$, $\boxed{}$

(iii) $\int \frac{dx}{1+\cos x} dx$ (hint: multiply by 1),

(iv) $\int_0^3 \frac{x^2\, dx}{\sqrt{9-x^2}}$,

(v) $\int_2^\infty \frac{dx}{(x-1)(x+1)}$,

(3) **(2 pts each, 10 pts total)**

(i) Expand using partial fractions: $\frac{x-1}{(x+1)^3}$.

(ii) Expand using partial fractions: $\frac{-2x+4}{(x^2+1)(x-1)^2}$.

For each of the following improper integrals, state whether the integral converges or diverges and explain why.

(iii) $\int_1^\infty x^2 e^{-x} dx$

(iv) $\int_0^1 \frac{x}{(1-x)^2} dx$

(v) $\int_0^\infty \frac{dx}{(1+x)\sqrt{x}}$

(4) **(5 pts each, 10 pts total)**: Do **TWO** of the following (your choice). Put your solutions on the following blank pages. **Explain all your work.** Put an "X" in the boxes corresponding to the two problems you want graded (choose at most 2):

(i) ☐ For which values of $0 < p < \infty$ does the integral $\int_0^\infty \frac{x^2}{x^p(1+x^p)} dx$ converge?

(ii) ☐ Use the definitions of cosh, sinh to prove $\cosh 2x = \cosh^2 x + \sinh^2 x$.

(iii) ☐ If $n > 0$ is an integer, evaluate $\int_0^{2\pi} [\sin(x) + \sin(2x) + \cdots + \sin(nx)]^2 dx$.

(iv) ☐ Which number is larger: 123456^{123457} or 123457^{123456}?