1. (2 pt each, 40 pts total) Place the letter corresponding to the correct answer in the box next to each question. Each correct answer is worth 2 points.

(i) The function \( f(x) = x^2 - x + 1 \) is (a) never negative (b) always negative (c) always increasing (d) always decreasing (e) concave down (f) none of these.

(ii) What is the maximum value of \( f(x) = x - 2x^4 \)? (a) 3/16 (b) 3/8 (c) 1/8 (d) 1/4 (e) 1/2 (f) none of these.

(iii) If a ball is dropped from 64 feet up, how fast is it going when it hits the ground (assume constant acceleration of 32 ft/sec^2)? (a) 16 ft/sec (b) 32 ft/sec (c) 48 ft/sec (d) 56 ft/sec (e) 64 ft/sec (f) none of these.

(iv) Find the slope of \( y^2 + x^2 = y^4 - 2x \) at the point \((-2, 1)\) (a) 2 (b) 1 (c) 0 (d) -1 (e) -2 (f) none of these.

(v) The definition of \( f'(x) \) is (a) \( \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \) (b) \( \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \) (c) \( \lim_{x \to 0} \frac{f(x)-f(x)}{x} \) (d) \( \lim_{h \to x} \frac{f(x+h) - f(x)}{h} \) (e) \( \lim_{h \to 0} \frac{f(x+h)-f(x)}{x} \) (f) none of these.

(vi) Suppose \( g(t) = (x^2 + \sin(x))^3 \). Then \( g'(t) = (a) 3(2x + \cos(x))^2 \) (b) \( 3(2x^2 - \cos(x))^2(2x + \sin(x)) \) (c) \( (x^2 + \sin(x))^3(2x + \cos(x)) \) (d) \( 3(x^2 + \sin(x))^2(2x + \cos(x)) \) (e) \( 3(x^2 + \sin(x))^2(2x - \cos(x)) \) (f) none of these.

(vii) The base of a solid is the region in the first quadrant between the line \( y = x \) and the curve \( y = 2\sqrt{x} \). The cross sections (perpendicular to the x-axis) of the solid are squares whose bases stretch from the line to the curve. The volume of the solid is (a) \( \int_0^1 \frac{1}{2}(x - 2\sqrt{x})^2 \) (b) \( \int_0^1 (x - 2\sqrt{x})^2 \) (c) \( \int_0^2 (x - 2\sqrt{x})^2 \) (d) \( \int_0^1 \frac{1}{2}(x - 2\sqrt{x})dx \) (e) \( \int_0^2 (x - 2\sqrt{x})dx \) (f) none of these.
(viii) \[ \int_{x^{-2/3}}^{x} f'(x) x^2 \, dx = x^2. \] Suppose \( f(x) = (a) x \) \( (b) 3^{1/3} x^{2/3} \) \( (c) x^2 \) \( (d) 2x^2 \) \( (e) \) none of these.

(ix) \[ G(x) = \int_{0}^{x} \cos(t^2) \, dt. \] Suppose \( G'(x) = (a) \sin^2(x) \) \( (b) \cos(x^2)2x \) \( (c) -2x \sin(x^2) \) \( (d) \cos(x) 2x \) \( (e) -\sin(x)2x \) \( (f) \) none of these.

(x) Find the equation of the curve in the \( xy \)-plane that passes through the point \((1, 3)\) if its slope at \( x \) is always \( 3x^2 + 2 \). \( (a) y = x^3 + 2x \) \( (b) y = 5 \) \( (c) y = 3x^2 - 2 \) \( (d) y = x^3 + 2x - 4 \) \( (e) x = -1 \) \( (f) \) none of these.

(xi) If \( f(x) = x^3 - 3x \) then the iterative formula for Newton’s method becomes \( x_{n+1} = \) \( (a) x_n - x_n^3 - 3x_n \) \( (b) x_n/(3x_n - 3) \) \( (c) x_n - (x_n^3 - 3x_n)/(3x_n^2 - 3) \) \( (d) x_n - (3x_n^2 - 3)/(x_n^3 - 3x_n) \) \( (e) x_n - 3x_n^2 + 3 \) \( (f) \) none of these.

(xii) Suppose \( f(t) = 1. \) Then \( \int_{0}^{1} f(t/2) \, dt = (a) 0 \) \( (b) 1 \) \( (c) 2 \) \( (d) 3 \) \( (e) 4 \) \( (f) \) none of these.

(xiii) The error formula for the left hand sum with \( n \) subintervals (for monotonic functions) is \( E \leq \frac{(b - a)}{n} |f(b) - f(a)|. \) Using this formula, what is the minimum \( n \) needed to compute \( \int_{0}^{1} \sqrt{x^2 + 2x + 1} \, dx \) with error \( \leq .001? \) \( (a) 2000 \) \( (b) 20,000 \) \( (c) 1,600 \) \( (d) 16,000 \) \( (e) 400 \) \( (f) 4,000 \)

(xiv) Solve the intial value problem \( y' = \cos^2(t) \) and \( y(\pi) = 2 \) \( (a) y = \tan(x) - 1 \) \( (b) y = \cos^2(t) - 1/\sqrt{2} \) \( (c) y = \tan(t) + 1 \) \( (d) y = \sin(1/t) \) \( (e) y = \tan(t) + 1 \) \( (f) \) none of these.

(xv) Simpson’s approximation for \( \int_{0}^{\pi} \sin(x) \, dx \) with \( n = 6 \) is (recall \( y_k = f(x_k) \) where \( x_k = a + k(b - a)/n \)) \( (a) \frac{\pi}{6} [0 + 4 \sin(\pi/3) + 2 \sin(\pi/2) + 4 \sin(\pi/6) + 0] \) \( (b) \frac{\pi}{8} [0 + 4 \sin(\pi/8) + 2 \sin(\pi/4) + 4 \sin(\pi/6) + 0] \) \( (c) \frac{\pi}{12} [0 + 2 \sin(\pi/12) + 4 \sin(\pi/6) + 2 \sin(\pi/3) + 4 \sin(\pi/6) + 0] \) \( (d) \frac{\pi}{2} [0 + 2 \sin(\pi/2) + 2 \sin(\pi/6) + 2 \sin(\pi/3) + 2 \sin(\pi/6) + 0] \) \( (e) \frac{\pi}{6} [0 + \sin(\pi/6) + \sin(\pi/3) + \sin(\pi/2) + \sin(\pi/6) + 0] \) \( (f) \) none of these.

(xvi) Let \( F(t) = \int_{0}^{1} (1 + \sin^4(x))(1 - x) \, dx \) Then \( F \) takes its maximum value at \( (a) 1 \) \( (b) 0 \) \( (c) \pi/2 \) \( (d) \pi \) \( (e) 2\pi \) \( (f) \) none of these.

(xvii) Find the left hand sum approximation for \( \int_{0}^{1} x^3 \) with \( n = 4. \) \( (a) 1 \) \( (b) 1/2 \) \( (c) 0 \) \( (d) -1/4 \) \( (e) -1/2 \) \( (f) \) none of these.

(xviii) \( E = \frac{1}{12}(b-a)^3 \) \( \frac{1}{M} \) \( M \) is the error term for \( (a) \) left hand sums \( (b) \) right hand sums \( (c) \) Simpson’s rule \( (d) \) trapezoid rule \( (e) \) Calvleri’s rule \( (f) \) none of these.
(xix) The area of the region between the graphs of \( x^3 \) and \( x^4 \) with \( 0 \leq x \leq 1 \) is
(a) 1 (b) 1/2 (c) 1/4 (d) 1/20 (e) 1/24 (f) none of these.

(xx) What is the name of the following result: if \( f \) is continuous on an interval containing \( a \) and \( b \) and \( f(a) < y < f(b) \) then there is a \( c \) with \( a < c < b \) such that
\( f(c) = y \) (a) Rolle’s theorem (b) mean value theorem (c) Riemann’s theorem (d) min-max theorem (e) Simpson’s rule (f) intermediate value theorem

2. (1 pt each, 5 pts total) Compute the derivative of the following functions.

(i) \( f(x) = x^{1/2} + x^{-1/2}, \quad \frac{d}{dx} f(x) = \)

(ii) \( f(x) = (1 + x^3) \sin(x), \quad \frac{d}{dx} f(x) = \)

(iii) \( f(x) = \cos(x)/x, \quad \frac{d}{dx} f(x) = \)

(iv) \( f(x) = \tan(x^2), \quad \frac{d}{dx} f(x) = \)
3. (2 pts each, 10 pts total) Find each of the following definite integrals (you need not simplify answers)

(i) \( \int_{-1}^{1} (2x^4 - x^2 - 2) dx, \)

(ii) \( \int_{0}^{\pi/4} \sin(2x) dx, \)

(iii) \( \int_{0}^{1} (x^2 + 3)^5 dx, \)
(iv) \( \int_0^\pi \sin^2(t) \cos(t) \, dt \),

(v) \( \int_0^\pi \frac{\cos^2(x) - \sin^2(x)}{\cos(x) + \sin(x)} \, dx \),

4. (3 pts each; 9 pts total) In each of the following, give the answer as a definite integral. You SHOULD NOT evaluate the integral.

(i) The region bounded by \( y = x^3 \), \( y = 0 \) and \( x = 1 \) is rotated around the \( x \)-axis to give a region in 3-space. Find the volume of the region.

(ii) The region \( 0 \leq y \leq x^2 + \sin(x) \) with \( 1 \leq x \leq 2 \) is rotated around the \( y \)-axis to give a region in 3-space. Find the volume of the region.
(iii) Find the arclength of the graph of \( y = \sin(x) \) between \( x = 0 \) and \( x = \pi \).

5. (3 pts each, 6 pts total) Evaluate the following limits.

   (i) \( \lim_{n \to \infty} \int_1^n \frac{1}{x^p} \, dx \).

   (ii) \( \lim_{n \to \infty} \frac{1}{n} \sum_{k=0}^{n-1} \left( \frac{k}{n} \right)^2 \) (Hint: use an integral)
6. (10 pts) A rocket in space starts at rest and is accelerated at $8 \, m/sec^2$ for 100 seconds, coasts without acceleration for 50 seconds and then is accelerated at $16 \, m/sec^2$ for 50 seconds after which it coasts without acceleration.

Remember that acceleration is the derivative of velocity and that velocity is the derivative of position.

(i) (3 pts) On the first graph, draw the rocket’s velocity as a function of time as exactly as possible.

(ii) (3 pts) On the second graph, sketch the distance traveled as a function of time. (Note, the vertical units are kilometers = 1000 meters).

(iii) (2 pts) When is the speed of the rocket 1200 m/sec?

(iv) (2 pts) When will the rocket be 500 kilometers from its starting point?