FINAL
MAT 142
12/16/05

Name	Sec.

ID number	TA's name

1	2	3	4	5	total

THIS EXAM IS WORTH 80 POINTS. PUT ALL ANSWERS IN THE SPACE PROVIDED. ADDITIONAL BLANK PAGES ARE PROVIDED FOR SCRATCH WORK. NO NOTES, BOOKS, CALCULATORS ARE ALLOWED.

1. **(2 pts each, 40 pts total)** Place the letter corresponding to the correct answer in the box next to each question. Each correct answer is worth 2 points.

(i) ☐ Simplify $\ln(x^2e^{2x})$ **(a)** $2\ln(x)+2x$ **(b)** $4x\ln(x)$ **(c)** $2x\ln(x)$ **(d)** $2\ln(x)+x^2$ **(e)** $4x^2$ **(f)** none of these.

(ii) ☐ If $\frac{5x-3}{x^2-2x-3} = \frac{A}{x+1} + \frac{B}{x-3}$ then $A =$ **(a)** 1 **(b)** 2 **(c)** 3 **(d)** 4 **(e)** 5 **(f)** none of these.

(iii) ☐ With the substitution $x = 3\sin\theta$, the integral $\int \frac{x^2dx}{\sqrt{9-x^2}}$ becomes **(a)** $9\int \frac{d\theta}{\sin^3\theta}$ **(b)** $9\int \sin^2\theta d\theta$ **(c)** $9\int\sin^3\theta d\theta$ **(d)** $27\int(1-\sin\theta)d\theta$ **(e)** $27\int\sin^3\theta d\theta$ **(f)** none of these.

(iv) ☐ Which of the following improper integrals converges? **(a)** $\int_0^1 \frac{\ln x}{x}dx$ **(b)** $\int_0^1 x^{-3}dx$ **(c)** $\int_0^1 x^{-1/4}dx$ **(d)** $\int_1^\infty \frac{1}{\sqrt{x}}dx$ **(e)** $\int_1^\infty \frac{1}{x}dx$. **(f)** none of these.

(v) ☐ If the Taylor series $\sum_{n=1}^\infty c_n(x-2)^n$ converges at $x = 5$, then it must also converge at **(a)** -5 **(b)** -2 **(c)** -1 **(d)** 0 **(e)** 7 **(f)** none of these.

(vi) ☐ Evaluate $\int_0^2 \frac{2xdx}{x^2-5}$. **(a)** $\ln 2$ **(b)** $-\ln 2$ **(c)** 0 **(d)** $-\ln 5$ **(e)** $\ln 4$ **(f)** none of these.

(vii) ☐ What is the inverse of $y = e^{2x+1}$? **(a)** $y = \sqrt{\ln(x) - 1}$ **(b)** $y = \frac{1}{2}\ln(x/e)$ **(c)** $y = \ln(\sqrt{x})$ **(d)** $y = \frac{1}{2}(\ln(x)+1)$ **(e)** $y = \ln(2x+1)$ **(f)** none of these.

(viii) ☐ The formula for Euler's method of solving $y' = f(x,y)$ is **(a)** $y_{n+1} = y_n - f(x_n,y_n)\Delta x$ **(b)** $y_{n+1} = y_n + f(x_n,y_n)$ **(c)** $y_{n+1} = y_n + f(x_n,y_n)\Delta x$ **(d)** $y_{n+1} = y_n - f(x_n,y_n)$ **(e)** $y_{n+1} = y_n + f(x_n,y_n)(\Delta x)^2$ **(f)** none of these.

(ix) ☐ Find the solution of the differential equation $\frac{dy}{dx} = (1+y^2)e^x$. **(a)** $y = \tan x$ **(b)** $y = e^{\tan x}$ **(c)** $y = 1+\tan^2 x$ **(d)** $y = e^x$ **(e)** $y = \tan(e^x)$ **(f)** none of these.

(x) $\square$ In an oil refinery, a tank contains 2000 gallons of gasoline that initially has 100 pounds of additive disolved in it. Gasoline containing 2 pounds of additive per gallon is pumped into the tank at 40 gal/min and the well mixed solution is pumped out at 45 gal/min. If $y(t)$ is the amount of additive at time t then y satisfies which of the following differential equations: **(a)** $\frac{dy}{dt} = 40 - \frac{45y}{2000-5t}$ **(b)** $\frac{dy}{dt} = 80 - \frac{45y}{2000-5t}$ **(c)** $\frac{dy}{dt} = 40 - \frac{45y}{2000-45t}$ **(d)** $\frac{dy}{dt} = 80 - \frac{40y}{2000-45t}$ **(e)** $\frac{dy}{dt} = 40 - \frac{40y}{2000-5t}$ **(f)** none of these.

(xi) $\square$ Which of the following is a true identity for hyperbolic trig functions? **(a)** $\sinh 2x = 2\sinh x \cosh x$ **(b)** $\sinh^2 x + \cosh^2 x = 1$ **(c)** $\cosh 2x = \cosh x + \sinh x$ **(d)** $\cosh^2 x = \sinh^2 x$ **(e)** $\sinh^2 x = \frac{1}{2}\cosh 2x$ **(f)** none of these.

(xii) $\square$ The improper integral $\int_1^\infty (1+x^s)^{-2} dx$ converges if and only if **(a)** $s > 1$ **(b)** $s > 1/2$ **(c)** $s > 0$ **(d)** $s < -1$ **(e)** $s < 0$ **(f)** none of these.

(xiii) $\square$ Which of the following is a geometric series? **(a)** $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots$ **(b)** $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$ **(c)** $1 + \frac{1}{2} + \frac{1}{8} + \frac{6}{4} + \ldots$ **(d)** $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \ldots$ **(e)** $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$ **(f)** none of these.

(xiv) $\square$ The solution of the differential equation $y' = y^2 - y - 2$ with initial condition $y(0) = 0$ **(a)** is constant **(b)** decreases to -1 **(c)** decreases to $-\infty$ **(d)** increases to 2 **(e)** increases to $+\infty$ **(f)** none of these.

(xv) $\square$ The sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots, \frac{n}{n+1}, \ldots$ is **(a)** non-decreasing and convergent **(b)** non-increasing and convergent **(c)** non-decreasing and divergent **(d)** non-increasing and divergent **(e)** bounded and divergent **(f)** none of these.

(xvi) $\square$ Which of the following sequences is not bounded for $n = 1, 2, 3, \ldots$? **(a)** $\{n^2 2^{-n}\}$ **(b)** $\{\sqrt{n+1} - \sqrt{n}\}$ **(c)** $\{n^2/(1+n^3)\}$ **(d)** $\{n/\ln n\}$ **(e)** $\{n^2/(1+n)^2\}$ **(f)** none of these.

(xvii) $\square$ Suppose $a_n = 2a_{n-1} + 1$. If $a_1 = 1$ then $a_5 =$ **(a)** 15 **(b)** 5 **(c)** 31 **(d)** 8 **(e)** 16 **(f)** none of these.

(xviii) $\square$ The series $\sum_{n=1}^\infty (2x+1)^n$ converges exactly for **(a)** all x **(b)** $-1 < x < 0$ **(c)** $-1 < x < 1$ **(d)** $0 < x < 1$ **(e)** $0 < x < 1/2$ **(f)** none of these.

(xix) $\square$ The root test says that if $a_n \geq 0$ then $\sum_{n=0}^\infty a_n$ converges if **(a)** $\lim_n a_{n+1}/a_n < 1$ **(b)** $\lim_n a_n < 1$ **(c)** $\lim_n (a_n)^{1/n} < 1$ **(d)** $\lim_n a_n/a_{n+1} < 1$ **(e)** $\lim_n n a_n < 1$ **(f)** none of these.

(xx) $\square$ Evalute the improper integral $\int_{-\infty}^\infty \frac{dx}{1+x^2}$. **(a)** π **(b)** $\pi/2$ **(c)** 1 **(d)** 2π **(e)** 2 **(f)** none of these.

2. **(1 pt each, 10 pts total)** Match each function with its Taylor series expansion.

(i) ☐ $(1-x)^{-1}$

(ii) ☐ $\sin x$

(iii) ☐ $\ln(1+x)$

(iv) ☐ $(1+x)^{1/3}$

(v) ☐ e^x

(vi) ☐ $x^2 \cos x$

(vii) ☐ e^{x^2}

(viii) ☐ $\frac{\ln(1+x)}{1+x}$

(ix) ☐ $\cos x - \sin x$

(x) ☐ $\sqrt{1+x^2}$

A $1 + 2x + 4x^2 + 8x^3 + 16x^4 + \ldots$
B $2x + \frac{2}{3}x^3 + \frac{2}{5}x^5 + \ldots$
C $x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \ldots$
D $1 + x^2 + x^4 + x^6 + \ldots$
E $1 + \frac{1}{3}x - \frac{2}{9}x^2 + \frac{10}{27}x^3 - \frac{80}{81}x^4 + \ldots$
F $1 - x - \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 - \ldots$
G $1 + \frac{1}{2}x^2 - \frac{1}{4}x^4 + \frac{3}{8}x^6 - \frac{15}{16}x^8 + \ldots$
H $x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \frac{1}{7}x^7 + \ldots$
I $1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \ldots$
J $1 + x + x^2 + x^3 + x^4 + \ldots$
K $1 + 2x + x^2 + \frac{1}{3}x^3 + \frac{1}{12}x^4 + \frac{1}{60}x^5 + \ldots$
L $x^2 - \frac{1}{3}x^6 + \frac{1}{120}x^{10} - \ldots$
M $x - \frac{1}{6}x^3 + \frac{1}{120}x^5 + \ldots$
N $1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 - \ldots$
O $x - \frac{3}{2}x^2 + \frac{11}{6}x^3 - \frac{25}{12}x^4 + \ldots$
P $x^2 - \frac{1}{2}x^4 + \frac{1}{24}x^6 - \ldots$
Q $1 + x^2 + \frac{1}{2}x^4 + \frac{1}{6}x^6 + \ldots$
R $1 + \frac{1}{3}x + \frac{1}{9}x^2 + \frac{1}{27}x^3 + \ldots$
S $1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \ldots$
T $1 - x + x^2 - x^3 + x^4 - \ldots$
U $x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \ldots$
V $1 - \frac{1}{3}x^3 + \frac{1}{4}x^4 - \frac{1}{5}x^5 + \ldots$
W $x + x^3 + x^5 + x^7 + \ldots$
X $1 + \frac{1}{4}x^2 + \frac{1}{27}x^3 + \frac{1}{256}x^4 + \ldots$
Y $1 + \frac{1}{2}x + \frac{1}{4}x^2 + \frac{3}{8}x^3 + \ldots$
Z none of these

3. **(1 pt each, 10 pts total)** Label each series as either **A** (Absolutely convergent), or **C** (Conditionally convergent or **D** (Divergent). each of the following infinite series converges or diverges.

(i) ☐ $1 + 1 + 1 + 1 + \ldots$

(ii) ☐ $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

(iii) ☐ $1 - 1 + \frac{1}{2} - \frac{1}{2} + \frac{1}{3} - \frac{1}{3} + \ldots$

(iv) ☐ $\sum_{n=1}^{\infty} n^2 2^{-n}$

(v) ☐ $\sum_{n=1}^{\infty} \frac{n}{n^n}$

(vi) ☐ $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \ldots$

(vii) ☐ $\sum_{n=0}^{\infty} \frac{\cos(n)}{n^2}$

(viii) ☐ $\sum_{n=0}^{\infty} \frac{\cos(\pi n)}{n}$

(ix) ☐ $\sum_{n=1}^{\infty} (\ln n)^{-2}$

(x) ☐ $\sum_{n=0}^{\infty} \sin(n)$

4. **(1 pt each, 10 pts total)** Evaluate each derivative or find some integral and put the letter of the correct answer in the box.

(i) ☐ $\frac{d}{dx}\ln x$

(ii) ☐ $\frac{d}{dx}\arcsin(x)$

(iii) ☐ $\frac{d}{dx}x^{\sin x}$

(iv) ☐ $\frac{d}{dx}\cosh^{-1}(x)$

(v) ☐ $\frac{d}{dx}\cosh x$

(vi) ☐ $\int \frac{dx}{\sqrt{3-4x^2}}$

(vii) ☐ $\int \sec x dx$

(viii) ☐ $\int x\cos x dx$

(ix) ☐ $\int \sin^3 x\cos^2 x dx$

(x) ☐ $\int \frac{dx}{1+x^2}$

A $x^{\sin x}(\frac{1}{x}\sin x + \ln x\cos x)$
B $\frac{1}{1-x^2}$
C $\frac{1}{x}$
D $-\sinh x$
E $\tan(x)$
F $x^{\sin x}\ln x\cos x$
G e^x
H $\ln|\sec x + \tan x|$
I $x\cos x + \sin x$
J $\frac{1}{2}\arcsin(\frac{2x}{\sqrt{3}})$
K $x\ln x$
L $\ln|\cos x + \tan x|$
M $1/\sqrt{1-x^2}$
N $1/(|x|\sqrt{x^2-1})$
O $\frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x$
P $\sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x$
Q $\cos^5 x - \sin^3 x$
R $\cosh x$
S $\cot x$
T $\tanh x$
U $\frac{1}{\sqrt{x^2-1}}$
V $\arcsin(x)$
W $x^{\cos x}$
X $x\sin x + \cos x$
Y $\sinh x$
Z none of these

5. **(5 pts each, 10 pts total)** Do TWO of the following problems (your choice). Put a mark in the box next to the two problems you want to be graded. Put your work on the following pages and clearly mark which problem you are doing.

(i) ☐ Solve the following differential equation: $(x+1)\frac{dy}{dx} - 2(x^2+x)y = e^{x^2}/(x+1)$.

(ii) ☐ State Taylor's theorem (or the remainder estimation theorem) and use it to prove that the Maclaurin series for $\sinh x$ converges to $\sinh x$ for all real numbers.

(iii) ☐ Find a power series solution (up to and including the x^4 term) to the differential equation $y'' - y' - y = 0$ with $y'(0) = 1$ and $y(0) = 1$

(iv) ☐ Give an example of a series so that $\sum_{n=1}^{\infty} a_n$ converges but such that $\sum_{n=1}^{\infty}(a_n)^2$ diverges.

(v) ☐ Prove $e = \sum_{n=0}^{\infty}\frac{1}{n!}$ is irrational.