SAMPLE FINAL MAT 142
FINAL is Friday, December 16, 2005,
11:30 to 1:00 in Physics P-112

1. Place the letter corresponding to the correct answer in the box next to each question.

(i) ☐ The sequence $\{a_n\} = \{1 + (-1)^n \frac{1}{n}$ converges to
(a) 0 **(b)** -1 **(c)** 1 **(d)** $\frac{1}{2}$ **(e)** $-\frac{1}{2}$ **(f)** it diverges

(ii) ☐ The sequence $\{a_n\} = \{(-1)^n(1 - \frac{1}{n})\}$ has least upper bound equal to
(a) -1 **(b)** 0 **(c)** 1 **(d)** 2 **(e)** $\frac{1}{2}$ **(f)** it has no upper bound

(iii) ☐ Define a sequence by $a_0 = 1$, $a_n = \frac{3}{2}a_{n-1}$. Then the sequence converges to
(a) 0 **(b)** 1 **(c)** 2 **(d)** 4 **(e)** $\frac{3}{2}$ **(f)** the sequence diverges

(iv) ☐ The infinite series $\sum_{n=0}^{\infty} 3^{-n}$ converges to
(a) 0 **(b)** $\frac{1}{3}$ **(c)** $\frac{2}{3}$ **(d)** $\frac{3}{4}$ **(e)** 1 **(f)** none of these

(v) ☐ The infinite series $\sum_{n=0}^{\infty} \frac{2^n}{n!}$ converges to
(a) 0 **(b)** 1 **(c)** e **(d)** e^2 **(e)** 2 **(f)** none of these

(vi) ☐ What is the inverse function of $y = \sqrt{1 - x^2}$ on $(0, 1)$?
(a) $x^2 - 1$ **(b)** $\sqrt{1 - x^2}$ **(c)** $x^2 + 1$ **(d)** $\sqrt{1 + x^2}$ **(e)** $\sqrt{1 - x}$ **(f)** none of these

(vii) ☐ $\int_0^2 \frac{2x}{x^2 - 5} dx =$
(a) $\ln 2$ **(b)** $\ln 5$ **(c)** $-\ln 5$ **(d)** $-\ln 2$ **(e)** 0 **(f)** none of these

(viii) ☐ $\frac{d}{dx} 2^{x^2} =$
(a) 2^{x^2} **(b)** $2^{x^2} 2x$ **(c)** $2^{x^2} \ln 2$ **(d)** $2^{x^2} 2x \ln 2$ **(e)** $2^{x^2} x^2 \ln 2$ **(f)** none of these

(ix) ☐ Find the limit $\lim_{x \to 0^+} (1 + x)^{1/x}$.
(a) 0 **(b)** $1/e$ **(c)** 1 **(d)** e **(e)** ∞ **(f)** none of these

(x) ☐ What is $\frac{d}{dx} \sin^{-1}(x)$,$|x| < 1$?
(a) $x/\sqrt{1 + x^2}$ **(b)** $1/\sqrt{1 + x^2}$ **(c)** $1/\sqrt{1 - x^2}$ **(d)** $-1/\sqrt{1 - x^2}$ **(e)** $1/(|x|\sqrt{x^2 - 1})$ **(f)** none of these

2. Evaluate each of the following integrals. You may use the table of integrals at the end of the book.

   (i) $\int \sin^3(x)dx$
   (ii) $\int \frac{dx}{1+\sin 3x}$
   (iii) $\int \sqrt{x^2-1}dx$
   (iv) $\int \frac{dx}{\sqrt{4+x^2}}$
   (v) $\int \frac{\sqrt{x+2}}{x}dx$

3. State whether each series diverges or converges. Explain your answer.

   (i) $\sum_{n=1}^{\infty} \frac{\sin n}{n^3}$
   (ii) $\sum_{n=1}^{\infty} \frac{1}{1+n^2}$
   (iii) $\sum_{n=1}^{\infty} (-1)^n n^{1/2}$
   (iv) $\sum_{n=1}^{\infty} \frac{n^4}{2^n}$
   (v) $\sum_{n=1} (-1)^n \frac{n^2}{1+n^3}$

4. Solve each of the following differential equations.

   (i) $y' = e^{x-y}$
   (ii) $y' = 3x^2 e^{-y}$
   (iii) $xy' + 3y = \frac{\sin x}{x^2}, x > 0$
   (iv) $xy' + 2y = 1 - \frac{1}{x}, x > 0$
   (v) $2y'e^{x/2} + y$

5. Write out the Taylor series at $x = 0$ up to order x^4 for each of the following functions.

   (i) $\sin(x^2)$
   (ii) $e^x \cos(x)$
   (iii) $\frac{1+x^2}{1-x}$
   (iv) $\sqrt{1+x}$
   (v) $\sin^2(x)e^{x^2+1}(1-\cos(x))$

6. Write out the Maclaurin series for $\sin x$ and $\tan x$ up to the third power. Use these series to evaluate

$$\lim_{x\to 0} \frac{\sin x - \tan x}{x^3}.$$

7. Prove that

$$1 + \frac{1}{2}\cdot 2 + \frac{1}{4}\cdot 3 + \ldots + \frac{n}{2^{n-1}} + \ldots = 4.$$

8. Give the definition of the hyperbolic functions $\sinh x$ and $\cosh x$. Using these definitions, show $\sinh 2x = 2\sinh x \cosh x$.

9. Quote Taylor's theorem and use it to show the Taylor series for $\sin(x)$ converges to $\sin(x)$ for all real numbers.