SAMPLE MIDTERM 2, MAT 141, FALL 2000

The second midterm will be on Friday, November 17 at the usual class time (12:40pm) and place (P-118 in Physics building).

1. Place the letter corresponding to the correct answer in the box next to each question.

(i) ☐ **What is the slope of the curve given by** $x^3 + y^3 - 9xy = 0$ **at the point** $(x, y) = (2, 4)$**? (a)** 1 **(b)** $\frac{24}{30}$ **(c)** $\frac{3}{4}$ **(d)** $\frac{9}{18}$ **(e)** $\frac{6}{5}$ **(f) none of these.**

(ii) ☐ **Suppose** $f(x) = |x^2 - 2x|$**. The set of critical points of** f **is (a)** $\{0\}$ **(b)** $\{1\}$ **(c)** $\{0, 1, 2\}$ **(d)** $\{2\}$ **(e)** $\{0, 2\}$ **(f) none of these.**

(iii) ☐ **Suppose** $g'(x) = \sin^{1999}(x)$**. The absolute maximum of** g **on** $[0, 2\pi]$ **occurs at**
(a) 0 **(b)** $\pi/4$ **(c)** $\pi/2$ **(d)** π **(e)** 2π **(f) none of these.**

(iv) ☐ **Find** $\frac{dy}{dx}$ **at the point** $(3, 5)$ **if** $y^2 + y - 3 = x^3$**.**
(a) 3 **(b)** $75/7$ **(c)** $9/11$ **(d)** $27/11$ **(e)** 0 **(f) none of these.**

(v) ☐ **Find the linearization of** $f(x) = x^3 - x$ **at** $x = 1$**.**
(a) $L(x) = 2x$ **(b)** $L(x) = 2(x + 1)$ **(c)** $L(x) = -2(x - 1) + 1$ **(d)** $L(x) = 2x + 1$ **(e)** $L(x) = 2(x - 1)$ **(f) none of these.**

(vi) ☐ **Use differentials to estimate the change in the surface area of a cube** $S = 6x^2$ **when the edge length goes from** x_0 **to** $x_0 + dx$
(a) $6dx$ **(b)** $6x_0dx$ **(c)** $12x_0dx$ **(d)** $12dx$ **(e)** $18x_0dx$ **(f) none of these.**

(vii) ☐ **The formula for finding sucessive approximations in Newton's method is**
(a) $x_{n+1} = x_n + f(x_n)/f'(x_n)$ **(b)** $x_{n+1} = x_n - f(x_n)/f'(x_n)$ **(c)** $x_{n+1} = x_n + f'(x_n)/f(x_n)$ **(d)** $x_{n+1} = x_n - f'(x_n)/f(x_n)$ **(e)** $x_{n+1} = x_n - f(x_n)f'(x_n)$ **(f) none of these.**

(viii) ☐ **The solution of the inital value problem** $\frac{dy}{dx} = x + 1$**,** $y(2) = 3$ **is**
(a) $y = x + 1$ **(b)** $y = x^2 - x$ **(c)** $y = \frac{1}{2}x^2 + 2$ **(d)** $y = x^2 + x + 1$ **(e)** $y = \frac{1}{2}x^2 + x$ **(f) none of these.**

(ix) ☐ **Suppose** $f'(x) = x^2 \sin^{10}(x)$**. Then on the interval** $[0, \frac{1}{2}\pi]$ **the function** f **is**
(a) increasing and concave down (b) increasing and concave up (c) decreasing and concave down (d) decreasing and concave up (e) constant (f) none of these.

(x) ☐ **The function $f(x) = x^3 - 3x^2 + 1$ has a point of inflection at $x = ?$**
(a) -2 (b) -1 (c) 0 (d) 1 (e) 2 (f) none of these.

2. Find each of the following indefinite integrals

- **(i)** $\int x^3 - x^2 + 2dx$,
- **(ii)** $\int \sin(3x)dx$,
- **(iii)** $\int \cos(3x + 2)dx$,
- **(iv)** $\int \sin^4(t)\cos(t)dt$,
- **(v)** $\int t(t^2 + 1)^{1/2}dt$,

3. State the mean value theorem.

4. Suppose the second hand on a clock has length 20 cm. At what rate is the distance between the tip of second hand and the 12 o'clock mark changing when the second hand points to 3 o'clock?

5. Suppose it takes 2 hours to replace the drill bit while drilling for oil. A new drill bit digs quickly at first, but slows down with time. Suppose that in t hours it can drill though $f(t)$ feet of rock.

- **(i) Suppose the drill bit is used for T hours before being replaced. What is the average speed of drilling (including the 2 hours to install the bit)?**
- **(ii) Show that to maximize this average speed the bit should should be replaced after T hours of use where T satisfies $f'(T) = f(T)/(T + 2)$.**
- **(iii) If $f(t) = 100t/(t + 5)$ find this time T.**