SAMPLE FINAL MAT 125 Spring 2004
FINAL IS MONDAY, MAY 16, 11:00am-
1:30pm

room	sections
Gym	1-9,11-15
Dance Studio	10, ELC4

NO CALCULATORS OR NOTES ALLOWED ON THE FINAL. PLEASE BRING
YOUR STUDENT ID. Both rooms are in Sports complex.

1. Differentiate the following functions: (1) $x^{3/4}$ (2) $\tan(x)$ (3) 2^x (4)$x \ln x$ (5) $\sin(x)/\ln(x)$ (6) $\cos(\sin(x))$ (7) $(2+x^2)^{50}(1+x)^{100}$ (8) $\cos(x^2 \ln(x))$ (9) $\ln(\tan(e^x))$ (10) $x^{\sin(x)}$.

2. Place the letter corresponding to the correct answer in the box next to each question.

   (i) ☐ Simplify $\log_2(4x^2 2^x)$ (a) $\ln 2 + 2\ln x + x$ (b) $\log_2 2 + 2\log_2 x + x$ (c) $\ln 4 + 2\ln x + x\ln 2$ (d) $2 + 2\ln x + x$ (e) $2 + 2\log_2 x + x$ (f) none of these.

   (ii) ☐ The initial size of a bacteria colony is 1000. After 1 hour the bacteria count is 8000. Assuming exponential growth, the time it takes the colony to double in size is approximately (a) 5 minutes (b) 10 minutes (c) 15 minutes (d) 20 minutes (e) 30 minutes (f) none of these.

   (iii) ☐ Suppose $\sin(t) > 0$. Then at t the function cos must be (a) positive (b) negative (c) increasing (d) decreasing (e) zero (f) none of these.

   (iv) ☐ What is the domain of $\frac{1}{x^2-x}$? (a) all x (b) all x except 0 (c) all x except 0 and 1 (d) all $x > 0$ (e) $0 \le x \le 1$ (f) none of these.

   (v) ☐ Express the following function as an explicit formula: take a number and add 1 to it; then square the result and multiply by 4. (a) $f(x) = 4(x+1)^2$ (b) $f(x) = (4x+1)^2$ (c) $f(x) = 4x^2 + 1$ (d) $f(x) = (4x)^2 + 1$ (e) $f(x) = 4(x^2 + 1)$ (f) none of these.

   (vi) ☐ The minimum or maximum value of a quadratic function $f(x) = ax^2 + bx + c$ occurs at $x =$ (a) $-b/2a$ (b) $b/2a$ (c) $-b/a$ (d) b/a (e) a/b (f) none of these.

   (vii) ☐ A rectanglar box with a volume of 60 ft^3 has a square base. Find a function which models the surface area of the box as a function of x, the length of one side of the base. (a) $2x^2 + 4\frac{60}{x}$ (b) $x^2 + 240x$ (c) $2x^2 + 2\frac{60}{x^2}$ (d) $x^2 + \frac{60}{x}$ (e) $2x^2 + \frac{4}{x^2}$. (f) none of these.

   (viii) ☐ If $\sin(x) = 1/\sqrt{2}$ then $\tan(x) =$? (a) 0 (b) 1/2 (c) $1/\sqrt{2}$ (d) 1 (e) either -1 or 1 (f) none of these.

   (ix) ☐ Evaluate $\sec(\pi/3)$. (a) 0 (b) 1/2 (c) $\sqrt{3}/2$ (d) 1 (e) 2 (f) none of these.

(x) ☐ Suppose f is given by the following table. Estimate the derivative of f at $x = 1$
(a) 1 (b) 2 (c) 3 (d) 4 (e) 6 (f) none of these.

| x	0	.2	.4	.6	.8	1.0	1.2	1.4	1.6	1.8	2.0
f(x)	1	1.1	1.3	1.7	2.4	2.8	3.2	4.2	4.4	4.6	4.8

3. Answer each of the questions about the function f graphed below. Each box has unit size. In each question, list **ALL** the points a in $[-6, 6]$ so that

    (i) $\lim_{x \to a} f(x)$ exists but does not equal $f(a)$.

    (ii) f has a jump at a but $f(a)$ equals the left hand limit at a.

    (iii) What is $\lim_{x \to -3+} f(x)$?
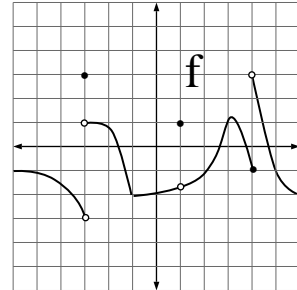
4. Answer each of the following T (for true) or F (for false).

    (i) f is increasing on the interval $(1, 2)$.

    (ii) f has a local maximum at $x = 2$.

    (iii) $f(4) > f(3)$.

    (iv) f is concave up on the interval $(3, 5)$.
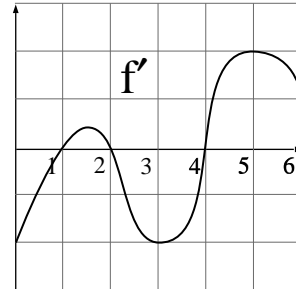
5. Compute the following using l'Hospital's rule. (i) $\lim_{x \to -1} \frac{x^2 - 1}{x + 1}$ (ii) $\lim_{x \to \infty} \frac{\ln(\ln(x))}{x}$. (iii) $\lim_{x \to 0+} x^{\sin(x)}$ (iv) $\lim_{x \to \infty} (\ln(x + 5) - \ln(x))$ (v) $\lim_{x \to 0} (\frac{1}{x} - \csc(x))$.

6. Find the line through $(3, 5)$ that cuts off the least area from the first quadrant.

7. At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/hr and ship B is sailing north at 25 k/hr. How fast is the distance between the ships changing at 4 pm?

8. If we use Newton's method with intial guess $x_1 = 1$ to solve the equation $x^3 - 3 = 0$, what are the next two guesses x_2 and x_3?