Math53: Ordinary Differential Equations
Winter 2004

Homework Assignment 2

Problem Set 2 is due by 2:15p.m. on Monday, 1/26, in 380Y

Problem Set 2:

PS2-Problem 1 (see next page); 4.1: 12,14; 4.2: 4; 4.3: 4,10,14,26; 4.4: 17 (1st part only); 4.5: 2,6,16,18,26,30,32,42.

Note 1: While the statement of Problem 1 looks long, most of it is actually a review.
Note 2: Since this problem set is due on a Monday and there is a number of problems from the preceding Friday, I will have office hours 4-6p.m. on Sunday, 1/25.

Daily Assignments:

Please review complex numbers, pp181-184, before F. 1/15

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Note: Problem 4.6: 13 is not part of Problem Set 2, but please do it as an exercise.
PS2-Problem 1

As discussed in class, if \( p \) and \( q \) are constants,

\[
(e^{(\lambda_2 - \lambda_1)t}(e^{-\lambda_2 t}y))' = e^{-\lambda_1 t}(y'' + py' + qy),
\]

(1)

if \( \lambda_1 \) and \( \lambda_2 \) are the two roots of the characteristic polynomial

\[
\lambda^2 + p\lambda + q = 0
\]

(2)

associated to the linear homogeneous second-order ODE

\[
y'' + py' + qy = 0.
\]

Thus, every second-order linear ODE with constant coefficients,

\[
y'' + py' + qy = f(t)
\]

(3)

can be solved in four steps:

Step 1: find the roots of the associated characteristic polynomial (2);
Step 2: multiply both sides of (3) by \( e^{-\lambda_1 t} \);
Step 3: use (1) to compress LHS of the resulting expression and to obtain

\[
(e^{(\lambda_2 - \lambda_1)t}(e^{-\lambda_2 t}y))' = e^{-\lambda_1 t}f(t);
\]

(4)

Step 4: solve (4) for \( y \) by integrating twice.

This approach mimics the integrating factor method for solving linear first-order ODEs, though it works only for constant \( p \) and \( q \). Its advantage over the methods described in Sections 4.3 and 4.5 of the text is that

(1) it works the same way whether or not \( \lambda_1 \) and \( \lambda_2 \) are distinct;
(2) it works the same way no matter what \( f \) looks like.

Use the above second-order integrating factor method to find the real (not complex) general solutions of

(a) \( y'' + 4y = 4\cos 2t \);
(b) \( y'' + 5y' + 4y = t \cdot e^{-t} \).

Compare your answers to (a) and (b) with your answers to 4.5:26 and 4.5:42, after you do them.

Note: When you work on 4.5:26 and 4.5:42, please use the methods requested in the textbook, as opposed to rewriting your solutions for (a) and (b).