

Math53: Ordinary Differential Equations Autumn 2004

Midterm II Information

Tuesday, 11/16, 2:15-3:05p.m.

General Information

This will be a closed-book, closed-notes exam. No calculators will be allowed. Midterm II will cover Chapters 5 and 7-9 of the textbook. Make sure you can do all problem set exercises from these chapters and perhaps some other related problems from the textbook. Please look in detail over these chapters, the lecture summaries handed out in class, and the solutions to the last three problem sets. Even if you know how to do every problem on the problem sets, you might find that the solutions describe a different way of doing some problems, which is quicker in some cases. You are very much encouraged to come to office and tutoring hours. *This midterm will be harder and longer than the first one.*

Pre-Midterm II Office and Tutoring Hours

Over the next few days, I will have office hours on Sunday 5-7 in 383N and Tuesday 9-12 in 383UU. Ziyu will hold office hours on Tuesday 12³⁰-2 in 380T, but *not* on Tuesday evening or Wednesday morning. These changes are for November 15-17 only. As always, Ken's office hours will be on Monday morning, 9-11, in 380U1, and the SUMO tutoring will be on Monday evening 6-10 in 381T.

After Midterm II

I expect to return the midterms to you on Wednesday in class. After that I will go over the most common errors and ways to try to avoid them. You may find this overview more helpful than that after the first midterm, even if you do very well on the second midterm.

I will be out of town from Thursday morning through Sunday night, November 18-21. Peter Storm will substitute for me for Thursday and Friday lectures. He will cover Chapter 6, *Numerical Methods*. While this topic is conceptually easier, in my view, than phase-plane sketches, I think it is best to see this explained with pictures and on examples.

There will be no office hours on Thursday morning. Ken will hold office hours on Sunday 5-7pm in 383N, instead of me. I will have office hours on Monday 9-11am in 383N, instead of Ken.

The last day to withdraw from the class is Sunday, 11/21. If you would like to discuss your standing in the class with me before that, we can do so either immediately after class on Wednesday or in my office, 383B, 4-6pm. On the other hand, if you do not pick up your exam on Wednesday, you will not be able to do so until Monday.

Background Material

You should be familiar with the most important facts from calculus and from the first third of this course.

Types of Problems to Expect

(1) *Laplace Transform and ODEs*: compute LT of functions directly from the definition; use tables of LTs, as in *Unit 3 Summary*, to compute LT and inverse LT of functions; use LT to solve IVPs involving high-order linear ODEs with constant coefficients. Examples: 5.1:1-29, 5.2:1-17,30-33,43, 5.3:1-36, 5.4:1-36, 5.5:1-25, and ODEs of high order.

(2) *Linear Algebra*: given a matrix, find its determinant, nullspace, a basis for the nullspace, inverse, eigenvalues, eigenvectors, generalized eigenvectors, and exponential; compute the product of the exponential of a matrix and of a vector (without necessarily computing the exponential first). Examples: 7.4:7-22; 7.5:1-40; 7.6:7-10,13-15,22-45,50-54; 9.1:1-12; 9.5:1-16,25-28.

(3) *Systems of Linear ODEs with Constant Coefficients*: find the general solution to planar and higher-dimensional systems, homogeneous and inhomogeneous; solve initial value problems; sketch phase-plane portraits for planar homogeneous systems. Examples: 9.2:1-12, 23-64 + phase-plane sketches; 9.4:7-11,21-26; 9.8:1-24.

(4) *Qualitative Descriptions*: determine whether the origin is a stable or unstable equilibrium; whether it is a nodal/spiral sink/source, etc. Examples: 9.6:1-14 and (3) above.

(5) *Laplace Transform, Convolution, and Delta Function*: compute convolution of two functions directly from the definition; use convolution to find inverse LT of a product of two functions; compute the unit impulse function for an ODE and use it to solve an inhomogeneous IVP. Examples: 5.6:2-7, 5.7:4-24,26-31.

(6) *Direction Fields, Component Plots, Phase-Plane Portraits*: direction fields for planar autonomous systems of first-order ODEs; given several component plots and phase-plane portraits, match each component plot with its phase-plane portrait. Examples: 8.2:13-21.

(7) *Solutions of Systems of ODEs, Higher-Order Equations*: given several functions and several ODEs or IVPs, match each function with the ODE, or IVP, it solves; rewrite a high-order ODE as a system of first-order ODEs. Examples: 8.1:7-16.

Remarks: (a) The primary focus of the midterm will be on (3) above, for planar systems, and on (1). The other things are closely related.

(b) The Laplace Transform tables that appear in *Unit 3 Summary* will be put on the inside front cover of the midterm.

(c) You do not have to memorize the exact expression for the real form of the general solution in the complex-eigenvalues case. However, you should have a good idea of what it looks like and what the corresponding phase-plane portrait looks like. You should also be able to solve IVPs involving a system of ODEs with complex eigenvalues using the complex form of the solution and reduce the answer to a real expression.