MAT 645: Pseudoholomorphic Curves
Spring 2022

Course Instructor

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*These OHs are intended primarily for MAT 127. Discussing MAT 645 at some other time, especially right after class, would be better.

Course Website

All updates, including schedule and references, will be posted on the course website,

`http://math.stonybrook.edu/~azinger/mat645-spr22`.

Please visit this website regularly.

Prerequisites

This course is limited to the PhD students in mathematics who have passed the comps. You also should have some understanding of complex geometry (MAT 545) and algebraic topology (MAT 541). Advanced PhD students from other departments must obtain permission from the instructor to enroll in this course.

Grading

Your grade will be based on class participation, in all possible forms.

Course Description

See next page.

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The aim of this course is to establish the foundations of J-holomorphic curves techniques in symplectic topology. These techniques, which intertwine the softness of topology and the rigidity of algebraic geometry, are central to modern symplectic topology, have found applications in algebraic geometry (in areas such as birational and enumerative geometry), and have connections to string theory. We will discuss the structure of J-holomorphic maps, removal of singularities, Gromov's compactness theorem, and Gromov-Witten invariants in special cases, covering roughly Chapters 1-7 and some appendices of

[MS04] D. McDuff and D. Salamon, *J-Holomorphic Curves and Symplectic Topology*, AMS Colloquium Publications 52, 2004/2012/2017.

We will mostly follow

[Z1] A. Zinger, *Notes on J-holomorphic maps.*
[Z2] A. Zinger, *Transversality for J-holomorphic maps: a complex-geometric perspective.*

The first provides a more systematic treatment of the structural and compactness aspects of the theory. The second is a more complex-geometric perspective on the regularity and invariants aspects, inline with the principle that pseudoholomorphic curves are a softer, more topological version of curves in complex/algebraic geometry.

Time-permitting, we will discuss such applications as Kontsevich's formula enumerating rational curves in $\mathbb{CP}^2$, Gromov's non-squeezing theorem, and other (non-)embedding theorems such as in

[Gr] M. Gromov, *Pseudoholomorphic curves in symplectic manifolds*, Invent. Math. 82 (1985), no. 2, 307–347.

[Gu] L. Guth, *Symplectic embeddings of polydisks*, Invent. Math. 172 (2008), no. 3, 477–489.

[MSc] D. McDuff and F. Schlenk, *The embedding capacity of 4-dimensional symplectic ellipsoids*, Ann. of Math. (2) 175 (2012), no. 3, 1191–1282.

The book [MS04] is a well-written, thorough introduction to J-holomorphic techniques. It involves quite a bit of analysis (Sobolev spaces, elliptic operators), but it is done only for the sake of specific geometric applications, not for its own sake. The organization of [MS04] also makes it easy to skip technical arguments, taking their conclusions on faith. Proofs of some basic facts in symplectic geometry not appearing in [MS04], such as Darboux's and Moser's Theorems, can be found in

[MS98] D. McDuff and D. Salamon, *Introduction to Symplectic Topology*, Oxford Mathematical Monographs, 1998/2017.

This book is more elementary, but barely touches on J-holomorphic maps. An even more elementary treatment of basic symplectic geometry appears in

[C] A. Cannas de Silva, *Lecture on Symplectic Geometry*, Lecture Notes in Mathematics 1764, Springer 2001.