MAT 531: Topology&Geometry, II
Spring 2006

Problem Set 2
Due on Thursday, 2/9, in class

Give concise, but complete, solutions. The entire problem set should not require more than a few pages. Please read the second version of *Notes on Vector Bundles* before starting on Problem 4.

1. Chapter 1, #10 (p51)
2. Chapter 1, #6 (p50)
3. Chapter 1, #7 (p50)
4. Suppose $k<n$. Show that the map

$$\iota\colon \mathbb{C}P^k \longrightarrow \mathbb{C}P^n, \qquad [X_0,\ldots,X_k] \longrightarrow [X_0,\ldots,X_k,\underbrace{0,\ldots,0}_{n-k}],$$

is a complex embedding (i.e. a smooth embedding that induces holomorphic maps between the charts that determine the complex structures on $\mathbb{C}P^k$ and $\mathbb{C}P^n$). Show that the normal bundle to the immersion, $\mathcal{N}_\iota$, is isomorphic to

$$(n-k)\gamma_k^* \equiv \underbrace{\gamma_k^* \oplus \ldots \oplus \gamma_k^*}_{n-k},$$

where $\gamma_k \longrightarrow \mathbb{C}P^k$ is the tautological line bundle (isomorphic as complex line bundles).
Hint: There are a number of ways of doing this, including:
(i) construct an isomorphism between the two vector bundles;
(ii) use Problems 4 and 5 from PS1 to determine transition data for $\mathcal{N}_\iota$ and $(n-k)\gamma_k^*$;
(iii) show that there exists a diffeomorphism between $(n-k)\gamma_k^*$ and a neighborhood of $\iota(\mathbb{C}P^k)$ in $\mathbb{C}P^n$ and that this implies that $\mathcal{N}_\iota=(n-k)\gamma_k^*$.

Final Exam

The current schedule for final exams is

Thur., 5/11, MAT 542; Fri., 5/12, MAT 550

Mon., 5/15, Calculus; Tues., 5/16, 11-1:30 MAT 535 and 2-4:30 MAT 531.

Given that the 531 final is immediately after the 535 final, as well as the day after the calculus finals, I'd like to suggest that the 531 final be moved to Monday, 5/8, or perhaps, to Tuesday, 5/9. The last class is on 5/4, and will likely be just review; I would have office hours the day before the final, even if that would be on a Sunday. If you are registered in this course, on 2/9 please turn in a separate sheet of paper with your thoughts on this, i.e. that you do not want to reschedule the final or list your preferences in order for the day and starting time of the exam (say, 10, 11, 12, 1, 2, and 3) if you do. It might be best if all of you could agree on the day and time of the final exam ahead before 2/9.