1. (a) For what values of \(t \in \mathbb{R}\), is the subspace 
\[ \{(x_1, \ldots, x_{n+1}) \in \mathbb{R}^{n+1}: x_1^2 + \ldots + x_n^2 - x_{n+1}^2 = t \} \]
a smooth embedded submanifold of \(\mathbb{R}^{n+1}\)?
(b) For such values of \(t\), determine the diffeomorphism type of this submanifold (i.e. show that it is diffeomorphic to something rather standard).

\textit{Hint:} Draw some pictures.

2. Show that the special unitary group
\[ SU_n = \{ A \in \text{Mat}_n \mathbb{C}: \bar{A}^t A = I_n, \det A = 1 \} \]
is a smooth compact manifold. What is its dimension?

3. (a) Suppose \(f: X \to M\) and \(g: Y \to M\) are smooth maps that are transverse to each other:
\[ T_{f(x)} M = \text{Im} \, df_x + \text{Im} \, dg_y \quad \forall (x, y) \in X \times Y \text{ s.t } f(x) = g(y). \]
Show that
\[ X \times_M Y \equiv \{(x, y) \in X \times Y: f(x) = g(y)\} \]
is a smooth (embedded) submanifold of \(X \times Y\).
(b) Suppose that \(f: X \to M\) is a smooth map and \(\pi: V \to M\) is a smooth vector bundle. The pullback of \(V\) by \(f\), \(\pi_1: f^*V \to X\), is the vector bundle defined by taking
\[ f^*V = \{(x, v) \in X \times V: f(x) = \pi(v)\} \subset X \times V. \]
In particular, \(f^*V\) is supposed to be a smooth manifold. Show that \(f^*V\) is in fact a smooth submanifold of \(X \times V\).

4. Chapter 1, #22 (p51)

5. Chapter 1, #17 (p51)

6. Let \(V\) be the vector field on \(\mathbb{R}^3\) given by
\[ V(x, y, z) = y \frac{\partial}{\partial x} - x \frac{\partial}{\partial y} + \frac{\partial}{\partial z}. \]
Explicitly describe and sketch the flow of \(V\).