Give concise, but complete, solutions. The entire problem set should not require more than a few pages.

1. Chapter 1, #2 (p50)

2. Suppose a group $G$ acts properly discontinuously on a smooth $n$-manifold $\tilde{M}$ by diffeomorphisms. Show that the quotient topological space $\tilde{M}/G$ admits a unique smooth structure such that the projection map $\tilde{M} \rightarrow M$ is a local diffeomorphism.

   **Note:** this implies that the circle, the infinite Mobius band, the Lens spaces (that are important in 3-manifold topology), the real projective space, and the tautological line bundle over it,$ 
   \begin{align*}
   S^1 &= \mathbb{R}/\mathbb{Z}, \quad s \sim s+1, \\
   MB &= (\mathbb{R} \times \mathbb{R})/\mathbb{Z}, \quad (s,t) \sim (s+1,-t), \\
   L(n,k) &= S^3/\mathbb{Z}_n, \quad (z_1,z_2) \sim (e^{2\pi i/n}z_1,e^{2\pi ik/n}z_2) \in \mathbb{C}^2, \\
   \mathbb{R}P^n &= S^n/\mathbb{Z}_2, \quad x \sim -x, \quad \gamma_n = (S^n \times \mathbb{R})/\mathbb{Z}_2, \quad (x,t) \sim (-x,-t),
   \end{align*}$

   are smooth manifolds in a natural way ($k$ and $n$ are relatively prime in the definition of $L(n,k)$).

3. (a) Show that the quotient topologies on $\mathbb{C}P^n$ given by $(\mathbb{C}^{n+1}-0)/\mathbb{C}^*$ and $S^{2n+1}/S^1$ are the same (i.e. the map $S^{2n+1}/S^1 \rightarrow (\mathbb{C}^{n+1}-0)/\mathbb{C}^*$ induced by inclusions is a homeomorphism).

   (b) Show that $\mathbb{C}P^n$ is a compact topological $2n$-manifold. Furthermore, it admits a structure of a complex (in fact, algebraic) $n$-manifold, i.e. it can be covered by charts whose overlap maps, $\varphi_\alpha \circ \varphi_\beta^{-1}$, are holomorphic maps between open subsets of $\mathbb{C}^n$ (and rational functions on $\mathbb{C}^n$).

   **Note:** you can do this with $n+1$ charts.

   (c) Show that $\mathbb{C}P^n$ contains $\mathbb{C}^n$, with its complex structure, as a dense open subset.

4. Chapter 1, #6 (p50) via 2nd suggested approach

5. Verify that the differential $d\psi$ of a smooth map $\psi: M \rightarrow N$, as defined in 1.22 (p16), is indeed well-defined. In other words, $d\psi(v)$ is a derivation on $T_{\psi(m)}$ for all $v \in T_m M$ and $m \in M$. 