Topology MAT 530 Homework 6

1. A covering $p : T \to X$ is called *regular* if the group $p_*\pi_1(T)$ is a normal subgroup in $\pi_1(X)$. Prove that p is regular if and only if no loop in X can be represented as the image of a loop in T and, at the same time, as the image of a path in T that is not a loop.

2. If $p : T \to X$ is a regular covering, then there is an action of the group $G = \pi_1(X)/p_*\pi_1(T)$ on T such that the orbits of this action coincide with the pre-images $p^{-1}(x)$, $x \in X$. This action is free, i.e. no non-identity element of G acts as the identity transformation.

3. Prove that any 2-fold covering is regular. Give an example of an irregular 3-fold covering.

4. Prove that the fundamental group of the torus $S^1 \times S^1$ is $\mathbb{Z} \times \mathbb{Z}$. More generally, the fundamental group of the direct product of two spaces is the product of their fundamental groups.

5. Let $p : T \to X$ be a covering. If X is simply connected (i.e. $\pi_1(X) = 0$), then p is a homeomorphism. (Recall that we always assume the spaces X and T to be path connected).

6*. Consider the polynomial $f(x) = x^3 + x$. Denote by Z the set of points where $f' = 0$ (this is just a pair of points). Prove that the restriction of f to $\mathbb{C} - Z$ is a covering over $\mathbb{C} - f(Z)$. Is this covering regular?