1. Prove that the map \( f_* : \pi_1(X, x_0) \to \pi_1(Y, y_0) \) induced by a continuous map \( f : (X, x_0) \to (Y, y_0) \), is a group homomorphism.

2. Prove that the fundamental group of the sphere \( S^n \) is trivial for \( n > 1 \).

3. Let \( A \) be a subspace of a topological space \( X \) and suppose that \( r : X \to A \) is a continuous map such that \( r(a) = a \) for all \( a \in A \). Such map is called a retraction of \( X \) onto \( A \). If \( a_0 \in A \), show that
\[
r_* : \pi_1(X, a_0) \to \pi_1(A, a_0)
\]
is surjective.

4. Let \( G \) be a topological group with operation \( \cdot \) and the identity element \( e \). Let \( \Omega(G, e) \) denote the set of all loops in \( G \) based at \( e \). If \( f, g \in \Omega(G, e) \), then define a loop \( f \otimes g \) be the rule
\[
(f \otimes g)(s) = f(s) \cdot g(s).
\]
Show that \( \Omega(G, e) \) is a group with respect to \( \otimes \) Show that \( \otimes \) induces a group operation on \( \pi_1(G, e) \).

5. In the notation of Problem 4, prove that \( \otimes \) coincides with the usual group operation on \( \pi_1(G, e) \). Prove that \( \pi_1(G, e) \) is commutative.

6. A subspace \( A \) of \( X \) is called a deformation retract of \( X \) if the identity self-map of \( X \) is homotopic to some retraction of \( X \) onto \( A \) so that during the homotopy every point of \( A \) remains fixed. Prove that if \( A \) is a path connected deformation retract of \( X \), then \( X \) is also path connected and \( \pi_1(X) = \pi_1(A) \).