MAT 530 Topology Final Exam

If you give complete solutions to at least 5 problems, then you will have the full grade for the exam.

1. Let R be the set of all real numbers with the countable complement topology, i.e. a subset $U \subseteq R$ is open if and only if $U = \varnothing$ or the complement $R - U$ is countable.

(a) Is R metrizable?

(b) Is R compact?

Explain your answer.

2. Let X be obtained by removing countably many lines from the Euclidean space $\mathbb{R}^3$. Show that X is connected.

3. Let $\mathbb{R}$ be the set of all real numbers with the usual topology and $\mathbb{R}^{\mathbb{N}}$ the infinite product of countably many copies of $\mathbb{R}$ with the product topology (Tychonoff topology). Does $\mathbb{R}^{\mathbb{N}}$ have a countable dense subset? Explain your answer.

4. Prove: a map $f : X \to Y$ between metric spaces is continuous if and only if $f(x_n) \to f(x)$ whenever $x_n \to x$ is a convergent sequence in X. (Recall that a map is said to be *continuous* if the preimage of any open subset is open).

5. Let X be the union of the unit circle centered at 0 and the segment between points $(1, 0)$ and $(2, 0)$. What is a universal covering of X? Compute the fundamental group of X.

6. How many coverings over $\mathbb{R}\mathrm{P}^2$ are there up to equivalence? Explain.

7. Is it true that any continuous map of a figure 8 (bouquet of two circles) to itself has a fixed point?