

MAT 320: Introduction to Analysis, Spring 2019

Homework Assignment 7

Please read the posted notes and p171mid-p179 (except Example 8b) from Ross's textbook thoroughly before starting on the problem set below.

Optional supplementary reading: Chapter 2 of Rudin's book

Problem Set 7 (**due at the start of recitation on Wednesday, 4/3**): Problems H-L (below)

Problem H

Let (X, d) be a metric space and $A, B \subset X$ be disjoint closed subsets. Show that there exist open subsets $U, V \subset X$ such that $A \subset U$, $B \subset V$, and $\overline{U} \cap \overline{V} = \emptyset$.

Hint: first use open balls around points of A and B to construct disjoint U and V and show that their closures are disjoint from B and A , respectively.

Problem I

- (a) Let (X, d) be a metric space, $U \subset X$ be an open dense subset, and $B \subset X$ be a nowhere dense subset. Show that the subset $U - \overline{B} \subset X$ is open and dense.

Let d be a square, round, or sum metric on \mathbb{R}^n .

- (b) Show that $\mathbb{Q}^n \subset \mathbb{R}^n$ is not the intersection of countably many open subsets of \mathbb{R}^n .
- (c) Show that $\mathbb{R}^n - \mathbb{Q}^n$ is not the union of countably many closed subsets of \mathbb{R}^n .
- (d) Give an example of *disjoint* subsets Y_1, Y_2, \dots of \mathbb{R}^n each of which is of the second category.

Problem J

Let (X, d) be a metric space and $Y \subset X$; thus, Y inherits a metric $d_Y \equiv d|_{Y \times Y}$ from (X, d) .

- (a) For a subset $A \subset Y$, let $\text{Cl}_X(A) \subset X$ and $\text{Cl}_Y(A) \subset Y$ denote the closures of A in (X, d) and in (Y, d_Y) , respectively. Show that

$$\text{Cl}_Y(A) = \text{Cl}_X(A) \cap Y.$$

- (b) Suppose (X, d) is a Baire space, i.e. satisfies the property of Ross's Theorem 21.7a or equivalently of 21.7b, and Y is open in (X, d) . Show that (Y, d_Y) is also a Baire space.

Problem K

Let (X, d) be a connected metric space containing at least two points. Show that X is uncountable.

Hint: Look at sets of the form $B_\delta(x)$ and $X - \overline{B}_\delta(x)$.

Problem L

Suppose (X, d) is a metric space, $A \subset X$ is a connected subset, and $A \subset B \subset \overline{A}$. Show that B is also connected.