MAT 312/AMS 351: Applied Algebra
Homework Assignment 9

Written Assignment due before 11:30am, Tuesday, 11/19

Please read Sections 4.4 and 6.1 before starting on the problem set.

Practice Problems (do not hand in; answers in the book): 4.4 1,2-4,6-10; 6.1 1-3*
*The solution to 6.1 3(i) should not include $1\pm\mathfrak{i}$, since this part is asking for *real* roots (unlike (ii))

Written Assignment: 4.4 2,5,11; Problem E below
Show your work; correct answers without explanation will receive no credit, unless noted otherwise

Please write your solutions legibly; the grader will disregard solutions that he does not find readily readable (you are encouraged to type up your solutions, especially if your handwriting is not absolutely immaculate). The problems on your solutions must appear in the assigned order; out-of-order problems will not be graded. All solutions must be stapled (no paper clips) and have your name (first name first), recitation number (R01 or R02), and HW number in the upper-right corner of the first page; otherwise, you may receive no credit.

NO late homework will be accepted

Problem E

Let $(R,+,\cdot)$ be a commutative ring with additive identity 0 and multiplicative identity 1. An element $u\in R$ is called a unit if it has a multiplicative inverse (thus, 0 is not a unit, and every nonzero element of a field is a unit).

(a) Show that the sets of powers series and polynomials with coefficients in R,

$$R[[x]] \equiv \left\{ \sum_{n=0}^{\infty} a_n x^n : a_0, a_1, \ldots \in R \right\} \qquad \text{and}$$
$$R[x] \equiv \left\{ \sum_{n=0}^{\infty} a_n x^n \in R[[x]] : \exists\, d\in\mathbb{Z}^{\geq 0} \text{ s.t. } a_n = 0 \ \forall\, n > d \right\},$$

have natural commutative ring structures. Specify the addition and product operations, additive identity $\mathbf{0}$, and multiplicative identity $\mathbf{1}$. Verify the required properties.

(b) Show that $a(x)\equiv 1+x$ is not a unit in $R[x]$.

(c) Show that $a(x)\equiv\sum_{n=0}^{\infty} a_n x^n$ is a unit in $R[[x]]$ if and only if a_0 is a unit in R.