MAT 312/AMS 351: Applied Algebra
Homework Assignment 11

Written Assignment due before 11:30am, Thursday, 12/5

Please read Sections 6.4 and 6.5 before starting on the problem set.

Practice Problems (do not hand in; answers in the book): 6.4 1-3; 6.5 1-5

Written Assignment: Problems I-K below
Show your work; correct answers without explanation will receive no credit, unless noted otherwise

Please write your solutions legibly; the grader will disregard solutions that he does not find readily readable (you are encouraged to type up your solutions, especially if your handwriting is not absolutely immaculate). The problems on your solutions must appear in the assigned order; out-of-order problems will not be graded. All solutions must be stapled (no paper clips) and have your name (first name first), recitation number (R01 or R02), and HW number in the upper-right corner of the first page; otherwise, you may receive no credit.

NO late homework will be accepted

Problem I

Let F be a field and $p \in F[x]$ be a polynomial of positive degree. Show that the ring $F[x]/(p)$ of polynomial congruence classes of p is a field if and only if p is irreducible.
Note: It is shown on p280 that $F[x]/(p)$ is a ring, even if F is just a ring; do not check this again.

Problem J

Let F be a field. A polynomial $p \in F[x]$ of positive degree d is called primitive if the remainders of the monomials x^i, $i = 1, 2, \ldots$, from dividing by p include every nonzero polynomial of degree less than d. Show that

(a) a primitive polynomial is prime;

(b) $1+x+x^2+x^3+x^4 \in \mathbb{Z}_2[x]$ is prime, but not primitive;

(c) the smallest $n \in \mathbb{Z}^+$ such that a primitive degree d polynomial $p \in \mathbb{Z}_2[x]$ divides x^n-1 is 2^d-1.

Hint for (a): Show first that $F[x]/(p)$ has no zero divisors if p is primitive and then use Problem I. For the latter, treat the cases F is finite and infinite separately. Feel free to ignore this hint. Maybe try (b) first.

Problem K

Let $f\colon \mathbb{Z}_2^4 \longrightarrow \mathbb{Z}_2^7$ be the cyclic code generated by the polynomial $p(x) = 1+x+x^3$.

(a) Show that this code corrects one error.

(b) Find the parity polynomial $q(x)$ for $p(x)$.

(c) The message received, possibly with an error, is 0110110. What message (codeword) was sent? What word does this codeword stand for?