

# MAT 312/AMS 351: Applied Algebra

## Homework Assignment 10

Written Assignment due before 11:30am, Tuesday, 11/26

Please read Sections 6.2 and 6.3 before starting on the problem set.

*Practice Problems (do not hand in; answers in the book):* 6.2 1-4; 6.3 1-6

**Written Assignment:** Problems F-H below

*Show your work; correct answers without explanation will receive no credit, unless noted otherwise*

*Please write your solutions legibly; the grader will disregard solutions that he does not find readily readable (you are encouraged to type up your solutions, especially if your handwriting is not absolutely immaculate). The problems on your solutions must appear in the assigned order; out-of-order problems will not be graded. All solutions must be stapled (no paper clips) and have your name (first name first), recitation number (R01 or R02), and HW number in the upper-right corner of the first page; otherwise, you may receive no credit.*

<b>NO late homework will be accepted</b>
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### Problem F

Factor the following polynomials into irreducible ones (and show that the factors are indeed irreducible).

(a)  $x^3+x+1$  in  $\mathbb{Z}_2[x]$     (b)  $x^2-3x-3$  in  $\mathbb{Z}_5[x]$     (c)  $x^2+1$  in  $\mathbb{Z}_7[x]$

### Problem G

Find a greatest common divisor of  $x^3-6x^2+x+4$  and  $x^5-6x+1$  in  $\mathbb{R}[x]$ .

### Problem H

Let  $F$  be a field (possibly finite). Show that there are infinitely many irreducible monic polynomials in  $F[x]$  (monic means that the coefficient of the highest power of  $x$  is 1).

*Hint:* How was a similar result proved for  $\mathbb{Z}$ ?