MAT 312/AMS 351 Midterm II

November 5, 2019 11:30am-12:50pm

Name:______________________ **ID:**______________

first name *first*

Recitation: R01 R02 (circle yours)

Tu 1-1:53pm W 11-11:53am

DO NOT OPEN THIS EXAM YET

Instructions

(1) Fill in your name and Stony Brook ID number and circle your recitation number at the top of this cover sheet.

(2) This exam is closed-book and closed-notes; no calculators, no phones.

(3) Please write legibly to receive credit. Circle or box your final answers. If your solution to a problem does not fit on the page on which the problem is stated, please indicate on that page where in the exam to find (the rest of) your solution.

(4) You may continue your solutions on additional sheets of paper provided by the proctor. If you do so, please write your name and ID number at the top of each of them and staple them to the back of the exam (stapler available); otherwise, these sheets may get lost.

(5) Anything handed in will be graded; incorrect statements will be penalized even if they are in addition to complete and correct solutions. If you do not want something graded, please erase it or cross it out.

(6) Show your work; correct answers only will receive only partial credit (unless noted otherwise).

(7) Be careful to avoid making grievous errors that are subject to heavy penalties.

(8) If you need more blank paper, ask a proctor.

Out of fairness to others, please **stop working and close the exam as soon as the time is called**. A significant number of points will be taken off your exam score if you continue working after the time is called. You will be given a two-minute warning before the end.

1 (10pts)	2 (12pts)	3 (10pts)	4 (20pts)	5 (18pts)	Tot (70pts)

Problem 1 (10pts)

Answer Only: circle **T** for TRUE and **F** for FALSE.

(a) The set $\mathbb{Z}^+$ of positive integers is a group under addition $+$. **T** **F**

(b) The set $4\mathbb{Z}$ of multiples of 4 is a group under addition $+$. **T** **F**

(c) The set $\mathbb{Q}-\{0\}$ of nonzero rational numbers is a group under multiplication $*$. **T** **F**

(d) The set of $n\times n$ invertible matrices is a group under component addition $+$. **T** **F**

(e) The set of $n\times n$ matrices is a group under matrix multiplication. **T** **F**

(f) The groups $(\mathbb{Z}_2,+)\times(\mathbb{Z}_2,+)$ and $(\mathbb{Z}_4,+)$ are isomorphic. **T** **F**

(g) Every group of order 5 is abelian. **T** **F**

(h) Every group of order 6 is abelian. **T** **F**

(i) Every group of order 7 contains an element of order 7. **T** **F**

(j) If a, b are elements of a group G, then $(ab)^{-1} = a^{-1}b^{-1}$. **T** **F**

Problem 2 (12pts)

Answer Only: only the answers appearing in the boxes provided below will be evaluated.

Let $\pi, \sigma \in S_5$ be the permutations

$$\pi = (123)(45) \qquad \text{and} \qquad \sigma = (12)(345)\,.$$

(a) Determine the permutations $\pi\sigma$ and $\sigma\pi$ in the two-row notation.

$$\boxed{\pi\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ & & & & \end{pmatrix}} \qquad \boxed{\sigma\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ & & & & \end{pmatrix}}$$

(b) Decompose $\pi\sigma$ and $\sigma\pi$ as products of disjoint cycles.

$$\boxed{\pi\sigma = \qquad\qquad} \qquad \boxed{\sigma\pi = \qquad\qquad}$$

(c) Determine the orders and signs of the permutations $\pi, \sigma, \pi\sigma, \sigma\pi$.

	π	σ	$\pi\sigma$	$\sigma\pi$
order				
sign				

Problem 3 (10pts)

Let $(G, *)$ be a group with identity element e.

(a) Let $a \in G$. Define what the order $\mathfrak{o}(a)$ of a in G means.

(b) Let $a \in G$. Show that $\mathfrak{o}(a^{-1}) = \mathfrak{o}(a)$.

(c) Let $a \in G$, $i \in \mathbb{Z}^+$, and $d = \gcd(i, \mathfrak{o}(a))$. Show that $\mathfrak{o}(a^i) = \mathfrak{o}(a)/d$. *Hint:* $\gcd(i, \mathfrak{o}(a)/d) = 1$.

Problem 4 (20pts)

Let $(G, *)$ be a group of order 9 with identity element e. Justify all answers below.

(a) Let $a \in G$ be any element. What are the possible orders $\mathfrak{o}(a)$ of a and why?

(b) Suppose G contains an element a with $\mathfrak{o}(a) = 9$. Show that $(G, *)$ is isomorphic to $(\mathbb{Z}_9, +)$.

(c) Suppose G contains no element a with $\mathfrak{o}(a) = 9$. Let $a, b \in G$ be such that $a, b \neq e$ and the cyclic subgroup $\langle a \rangle$ of G generated by a does not contain b. Show that

$$\mathfrak{o}(a), \mathfrak{o}(b) = 3 \qquad \text{and} \qquad G = \{e, a, a^2, b, b^2, ab, a^2b, ab^2, a^2b^2\}.$$

(d) Under the assumptions in (c), show that $ba \neq a^2b^2$. *Hint:* assume $ba = a^2b^2$ and compute $(ab)^2$.

(e) Under the assumptions in (c), show that $ba \neq a^2b$. *Hint:* assume $ba = a^2b$ and compute $(ab)^2$.

Problem 5 (18pts)

A linear coding function $f\colon \mathbb{Z}_2^3 \longrightarrow \mathbb{Z}_2^6$ is given by the generator matrix

$$G = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

(a) What codewords does f produce for 000 and 111? Answer only.

codeword for 000 =	codeword for 111 =

(b) What words do the codewords 001111 and 111010 stand for? Answer only.

001111 stands for	111010 stands for

(c) What is the maximum number of errors can this code detect? What is the maximum number of errors can this code correct? Put your answers in the boxes provided and justify them below.

max to detect =	max to correct =

(d) The messages received, possibly with errors, are (i) 110111 and (ii) 011100. What words should these messages be decoded to? Put your answers in the boxes provided and justify them below.

(i)	(ii)