MAT 127: Calculus C, Spring 2022
Homework Assignment 4

WebAssign Problems due before 9am, Wednesday, 02/23
20% bonus for submissions **before 9am, Saturday, 02/19**

Written Assignment due before 4pm, Wednesday, 02/23
in your instructor's office (L01 in Math 44-101B, L02/3 in Math 3-111)

Please read Sections 2.2,4.4 and *DE Notes* thoroughly before starting on the problem set; looking over Sections 7.4,7.5 of the WebAssign textbook may be helpful too.

Written Assignment: Problems D,E (below and next page)
Show your work; correct answers without explanation will receive no credit, unless noted otherwise.

Please write your solutions legibly; the graders will disregard solutions that are not readily readable. All solutions must be stapled (no paper clips) and have your name (first name first), lecture number (L01, L02, or L03), and HW number in the upper-right corner of the first page.

Problem D

(a) Use Euler's Formula to obtain the cosine/sine addition/subtraction formulas:

$$\cos(\alpha\pm\beta) = \cos\alpha\cdot\cos\beta \mp \sin\alpha\cdot\sin\beta \qquad \text{and} \qquad \sin(\alpha\pm\beta) = \sin\alpha\cdot\cos\beta \pm \cos\alpha\cdot\sin\beta.$$

(b) Use the last two formulas to obtain the cosine/sine double angle formulas:

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta \qquad \text{and} \qquad \sin 2\theta = 2\cos\theta\cdot\sin\theta.$$

Problem E

By Problem B on HW2, the first-order differential equation

$$y' - by = f(x), \quad y = y(x), \qquad b = const,$$

can be solved by multiplying both sides by e^{-bx}. This equation then becomes

$$\big(\mathrm{e}^{-bx}y\big)' = \mathrm{e}^{-bx}f(x)$$

and can be solved by integrating both sides. Note that b is the root of the associated linear equation $r-b=0$. This approach has an analogue for second-order *in*homogeneous linear equations

$$y'' + by' + cy = f(x), \quad y = y(x), \qquad b, c = const. \tag{1}$$

(a) If r_1, r_2 are the two roots of the quadratic equation $r^2+br+c=0$ associated to (1), show that

$$\big(\mathrm{e}^{(r_1-r_2)x}(\mathrm{e}^{-r_1x}y)'\big)' = \mathrm{e}^{-r_2x}(y''+by'+cy). \tag{2}$$

By (2), equation (1) is equivalent to

$$\big(\mathrm{e}^{(r_1-r_2)x}(\mathrm{e}^{-r_1x}y)'\big)' = \mathrm{e}^{-r_2x}f(x), \quad y = y(x), \tag{3}$$

which can be solved by integrating twice.

(b) Find the general solution $y = y(x)$ to the differential equation

$$y'' + 5y' + 4y = \mathrm{e}^{-x}.$$

Hint 1: comparing this equation with equation (1) above, what are b, c, $f(x)$, r_1, and r_2 here? How does the sentence following part (a) apply in this case?

Hint 2: choosing the order of the roots wisely could simplify the computation.

(c) Find the general solution $y = y(x)$ to the differential equation

$$y'' + 4y = 4\cos 2x.$$

Hint 1: see the two hints above.

Hint 2: replacing $\cos 2x$ by $\mathrm{e}^{2\mathrm{i}x}$ and then taking the real part of the resulting general solution would simplify the computation. This real part is the general (real) solution to the above equation because $\cos 2x$ is the real part of $\mathrm{e}^{2\mathrm{i}x}$ and all coefficients in the equation are real.

Note: If you ask someone at MLC/RTC to help you with this problem, do not just point them to part (b) or (c), but ask them to read the introduction at the beginning of the problem. They may not know how to help you right away because an approach to more general equations of this form is introduced in MAT 303. The approach described above is simpler, but is applicable to a narrower set of cases.