WebAssign Problems due before 9am, Wednesday, 11/17 (all sections)
20% bonus for submissions before 9am, Saturday, 11/13

Written Assignment due before
9:35am Wednesday, 11/17 in Library E4320 if enrolled in L01
5:20pm Thursday, 11/18 in Library W4525 if enrolled in L02
2:20pm Thursday, 11/18 in Library W4540 if enrolled in L03

This problem set is a bit longer than usual, since it covers about 1.5 weeks.

Please read Sections 8.4 and 8.5 in the textbook thoroughly before starting on the corresponding problems below.

Written Assignment: 8.4 8,20,24,33; 8.5 24,26,31,34; Problem G (see below)
justify your answers on all problems

Please write your solutions legibly; the graders may disregard solutions that are not readily readable. All solutions must be stapled (no paper clips) and have your name and lecture number in the upper-right corner of the first page.

Problem G

(a) Show that the series

\[ g(z) = \sum_{n=1}^{\infty} \left( \frac{1}{z-n \pi} + \frac{1}{z+n \pi} \right) \]

converges for every \( z \neq m \pi \) for any nonzero integer \( m \) and that \( g(0)=0 \).

Hint: combine the fractions and use the Absolute Convergence Test.

(b) The function

\[ f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} \left( \frac{1}{z-n \pi} + \frac{1}{z+n \pi} \right) \]

is thus well-defined for every \( z \neq m \pi \) for any integer \( m \). Show that

\[ \lim_{z \to 0} zf(z) = 1, \quad f(-z) = -f(z), \quad f(z+\pi) = f(z), \quad f(\pi/2) = 0, \quad (1) \]

with the middle identities holding whenever either side is defined (\( z \neq m \pi \) for any integer \( m \)).

Hint: use partial sums for the third equality; the other three are easy.

(c) What is the “simplest” function that satisfies all identities in (1)? (answer only)

Note: This all leads to \( \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \) as stated in 8.3; see solutions for more details.