WebAssign Problem due before 9am, Tuesday, 10/05 (all sections)
20% bonus for submissions before 9am, Friday, 10/01

Written Assignment due before
5:30pm Tuesday, 10/05 in Math 3-121 if enrolled in L01
5:20pm Tuesday, 10/05 in Library W4525 if enrolled in L02
2:20pm Tuesday, 10/05 in Library W4540 if enrolled in L03

Note the earlier due dates (because of Midterm I); solutions will be available after 7pm on 10/05)

Please read Notes on Second-Order Differential Equations (called DE Notes below) thoroughly before starting on the problem set.

Written Assignment: DE Notes 1,7,12,16; Problem D (next page)
Show your work; correct answers without explanation will receive no credit, unless noted otherwise.

Please write your solutions legibly; the graders may disregard solutions that are not readily readable. All solutions must be stapled (no paper clips) and have your name and lecture number in the upper-right corner of the first page.
Problem D

By Problem B on HW2, the first-order differential equation

\[ y' - by = f(x), \quad y = y(x), \quad b = \text{const}, \]

can be solved by multiplying both sides by \( e^{-bx} \). This equation then becomes

\[ (e^{-bx}y)' = e^{-bx}f(x) \]

and can be solved by integrating both sides. Note that \( b \) is the root of the associated linear equation \( r - b = 0 \). This approach has an analogue for second-order inhomogeneous linear equations

\[ y'' + by' + cy = f(x), \quad y = y(x), \quad b, c = \text{const}. \]  

(1)

(a) If \( r_1, r_2 \) are the two roots of the quadratic equation \( r^2 + br + c = 0 \) associated to (1), show that

\[ (e^{(r_1-r_2)x}(e^{-r_1x}y)')' = e^{-r_2x}(y'' + by' + cy). \]  

(2)

By (2), equation (1) is equivalent to

\[ (e^{(r_1-r_2)x}(e^{-r_1x}y)')' = e^{-r_2x}f(x), \quad y = y(x), \]  

(3)

which can be solved by integrating twice.

(b) Find the general solution \( y = y(x) \) to the differential equation

\[ y'' + 5y' + 4y = e^{-x}. \]

Hint 1: comparing this equation with equation (1) above, what are \( b, c, f(x), r_1, \) and \( r_2 \) here? How does the sentence following part (a) apply in this case?

Hint 2: choosing the order of the roots wisely could simplify the computation.

(c) Find the general solution \( y = y(x) \) to the differential equation

\[ y'' + 4y = 4 \cos 2x. \]

Hint 1: see the two hints above.

Hint 2: replacing \( \cos 2x \) by \( e^{2ix} \) and then taking the real part of the resulting general solution would simplify the computation. This real part is the general (real) solution to the above equation because \( \cos 2x \) is the real part of \( e^{2ix} \) and all coefficients in the equation are real.

Note: If you ask someone at MLC/RTC to help you with this problem, do not just point them to part (b) or (c), but ask them to read the introduction at the beginning of the problem. They may not know how to help you right away because an approach to more general equations of this form is introduced in MAT 303. The approach described above is simpler, but is applicable to a narrower set of cases.