WEBASSIGN PROBLEMS DUE BEFORE 9AM, WEDNESDAY, 09/22 (ALL SECTIONS)  
20% BONUS FOR SUBMISSIONS BEFORE 9AM, SATURDAY, 09/18

WRITTEN ASSIGNMENT DUE BEFORE  
9:35AM WEDNESDAY, 09/22 IN LIBRARY E4320 IF ENROLLED IN L01  
5:20PM THURSDAY, 09/23 IN LIBRARY W4525 IF ENROLLED IN L02  
2:20PM THURSDAY, 09/23 IN LIBRARY W4540 IF ENROLLED IN L03

This problem set is longer than usual, since it covers about 1.5 weeks.

Please read Section 7.2 and the first half of Section 7.3 thoroughly before starting on the problem set.

**Written Assignment:** 7.2 12,18,21*; p551 5**; Problem B (next page)  
*use simple fractions, p/q; no rounding  
**hint: assume f is smooth for x ≠ 0; write the last condition as an equation and differentiate

Show your work; correct answers without explanation will receive no credit, unless noted otherwise.

Please write your solutions legibly; the graders may disregard solutions that are not readily readable. All solutions must be stapled (no paper clips) and have your name and lecture number (L01, L02, or L03) in the upper-right corner of the first page.
Problem B

The Fundamental Theorem of Calculus from Calculus B provides a quick way of finding the general solution to a differential equations of the form

\[ y' = f(x), \quad y = y(x). \]  

(1)

It turns out that every equation of the form

\[ y' + a(x)y = f(x), \quad y = y(x), \]  

(2)

can be reduced to (1). Simply multiply both sides of (2) by a nonzero function \( h = h(x) \) such that \( h' = ah \):

\[ h(x)y' + a(x)h(x)y = h(x)f(x) \iff hy' + h'y = hf \iff (hy)' = hf. \]

We can integrate both sides of the last equation and then divide by \( h \). For example, multiplying

\[ y' + 2xy = 2x, \quad y = y(x), \]  

(3)

by \( h(x) = e^{x^2} \) gives

\[ e^{x^2}y' + 2xe^{x^2}y = 2xe^{x^2} \iff (e^{x^2}y)' = 2xe^{x^2} \iff e^{x^2}y = \int 2xe^{x^2} \, dx = e^{x^2} + C. \]

So the general solution of the differential equation (3) is \( y(x) = 1 + Ce^{-x^2} \).

(a) Show that for any function \( a = a(x) \), there exists a nonzero function \( h = h(x) \) such that \( h' = ah \).

*Hint: see HW1 Problem A.*

(b) Find the general solution of the differential equation

\[ y' + 2y = 2e^x, \quad y = y(x). \]

What is the relation with the solution in Section 7.1, Exercise #1?

*Hint for 1st part: comparing this equation with equation (2) above, what are the functions \( a(x), f(x), \) and \( h(x) \) here? How does the sentence following equation (2) apply in this case?*

(c) Find the solution to the initial value problem

\[ y' = x + y, \quad y = y(x), \quad y(0) = 1. \]

What is the relation with the numbers in the last column of the table at the bottom of p504 in the book?