MAT 303 Assignment 1.
Hand in to the instructor in class on Monday, February 16.

**Problem 1.** In each case verify by substitution that the function is a solution of the corresponding differential equation

1) \( y(x) = \sin \left( \frac{x}{2} \right) - 2 \cos \left( \frac{x}{2} \right) \), \( 4y'' + y = 0 \),
   2) \( y(x) = e^{x^2}, \ y' = 2xy \),
   3) \( y(x) = \sqrt{x^2 + 1}, \ (y')^2 = 1 - \frac{1}{y^2} \).

**Problem 2.** Find the general solutions of the following differential equations:

1) \( \frac{dx}{dt} = 3t^2 + 2t - \cos(2t) \), \( 2) y' = x^2 \sin(x^3) \).

**Problem 3.** Solve the initial value problems:

1) \( \frac{dy}{dt} = \frac{t}{t^2 + 1}, \ y(0) = 5 \), \( 2) xy' = x^2 - 2, \ y(-1) = 0 \).

**Problem 4.** A car starting from rest reached the velocity 30 mi/h (44 ft/s) after traveling the distance of 44 ft. Assuming that the car had constant acceleration find this acceleration and the time which took the car to reach 60 mi/h.

**Problem 5.** Solve the following first order separable differential equations:

1) \( y' = x^2y \), \( y(2) = 1 \), \( 2) \frac{dx}{dt} = x + \frac{1}{x} \).

**Problem 6.** Among the following differential equations solve the one which is first order and separable

1) \( \frac{d^2x}{dt^2} = x^2t^2 \), \( 2) \frac{dy}{dt} = t^2 + y \sin t \),
   3) \( y' - 1 = xy + x + y \), \( 4) (y')^2 = x^2 + y^2 \).
Problem 7. Show by substitution that the formula

\[ y(x) = \frac{2}{1 + Ce^x} - 1, \]  

where \( C \) is a constant, gives a general solution of the differential equation

\[ 2y' = y^2 - 1. \]

Show that formula (1) is not the general solution of the given equation by finding a solution which is not described by (1).