THERE ARE TEN PROBLEMS. EACH PROBLEM HAS THE SAME VALUE 10 POINTS.

SHOW YOUR WORK

DO NOT TEAR-OFF ANY PAGE

NO CALCULATORS  NO CELLS ETC.

ON YOUR DESK: ONLY test, pen, pencil, eraser and student ID

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1. Solve the initial value problem

\[ x e^y y' - 2e^y = 3x^3, \quad y(1) = 0. \]
2. Show that the following differential equation is exact, then solve it:

\[(3y - \sin x)dy + (1 - y \cos x)dx = 0.\]
3.

Suppose that a motorboat is moving at $4\text{m/s}$ when its motor suddenly quits, and that 10 seconds later the velocity of the boat is $2\text{m/s}$. Assume that the resistance motorboat encounters is proportional to its velocity. Find the velocity of the boat in 20 seconds after the motor has quit.
4.

In a population $P(t)$ of rabbits both the birth and the death rates are proportional to $P^2(t)$. Initially, there were one thousand of rabbits. After 9 years only 800 of rabbits left. Find $P(t)$. 
5.

Solve the non-homogeneous equation

\[ y'' - 3y' + 2y = \sin x. \]
6.

Using the elimination method, solve the initial value problem:

$$X' = \begin{bmatrix} -t & 1 \\ -t^2 & t \end{bmatrix} X, \quad X(0) = \begin{bmatrix} 5 \\ -2 \end{bmatrix}.$$
7.

Show that the following three functions are linearly independent on the real line.

\[ X_1(t) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad X_2(t) = \begin{bmatrix} t \\ t^2 + t \\ 3t \end{bmatrix}, \quad X_3(t) = \begin{bmatrix} t + 1 \\ 0 \\ 1 \end{bmatrix}. \]
8.

Solve the system of differential equations

\[ X' = \begin{bmatrix} 3 & -1 & 0 \\ -2 & 2 & 0 \\ 0 & 2 & -1 \end{bmatrix} X. \]
Solve the initial value problem

\[ X' = \begin{bmatrix} 1 & -5 \\ 1 & 3 \end{bmatrix} X, \quad X(0) = \begin{bmatrix} 0 \\ 6 \end{bmatrix}. \]
10.

Solve the system of homogeneous equations

\[ X' = \begin{bmatrix} 3 & -4 \\ 3 & -5 \end{bmatrix} X. \]

Determine the type of the critical point at 0. Sketch the phase portrait.