MAT 303 Assignment 6.
Hand in to the instructor in class on Friday, December 7.

Problem 1. Solve the initial value problem

\[ x' = \frac{2}{t}x + \frac{y}{t}, \quad y' = (t - \frac{2}{t})x - \frac{y}{t}, \quad x(\pi) = 0, \quad y(\pi) = -2\pi \]

using the method of elimination.

Problem 2. Solve the nonhomogeneous system

\[ x_1' = -3x_1 + 4x_2 - 4e^t, \quad x_2' = 6x_1 - 5x_2 + 6e^t. \]

Problem 3. Prove that the trajectories of the system

\[ x' = y, \quad y' = x \]

are hyperbolas.

In Problems 4–5, given a matrix \( A \) and a vector \( X_0 \), solve the initial value problem

\[ X' = AX, \quad X(0) = X_0 \]

using the eigenvalue method.

Problem 4.

\[ A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & -3 & 0 \\ -1 & 5 & 2 \end{bmatrix}, \quad X_0 = \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}. \]

Problem 5.

\[ A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & -2 \\ 0 & 1 & 3 \end{bmatrix}, \quad X_0 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}. \]
In problems 6, 7 solve the system $X' = AX$, determine the type of the critical point 0 and sketch the phase portrait of the system.

**Problem 6.**

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}.$$

**Problem 7.**

$$A = \begin{bmatrix} 3 & 4 \\ 1 & 3 \end{bmatrix}.$$