Problem 1. Show directly that the functions
\[ f(x) = 3x - 2, \quad g(x) = 2x^2 - x, \quad h(x) = 3x^2 - 1 \]
are linearly dependent on the real line. That is, find constants \( c_1, c_2, c_3 \) (not all equal to zero) such that \( c_1 f(x) + c_2 g(x) + c_3 h(x) \) is identically equal to zero.

Problem 2. Using Wronskian, show that the functions
\[ f(x) = e^x, \quad g(x) = \sin x, \quad h(x) = x \sin x \]
are linearly independent on the real line.

Problem 3. 1) Verify that the functions \( y_1 = 1, \quad y_2 = x^3, \quad y_3 = \ln x \) are solutions of the differential equation
\[ x^2 y^{(3)} - 2y' = 0. \]
2) Show that \( y_1, y_2, y_3 \) are linearly independent.
3) Solve the initial value problem
\[ y(1) = 2, \quad y'(1) = 2, \quad y''(1) = 7. \]

Problem 4. Find the general solution of the differential equation
\[ y^{(4)} - \frac{3}{2} y'' + \frac{1}{2} y = 0. \]

Problem 5. Assume that a homogeneous differential equation with constant coefficients has the characteristic equation of the form
\[ (2r - 3)(r - 1)^2(r + 2)^2 = 0. \]
Using polynomial differential operators show that \( y = xe^{-2x} \) is a particular solution of this differential equation.

**Problem 6.** Solve the initial value problem

\[
y'' + 2y' + 5y = 0, \quad y(\pi) = 0, \quad y'(\pi) = 1.
\]

**Problem 7.** Assume that the roots of a characteristic polynomial of a homogeneous differential equation with constant coefficients are:

\[0, 0, 0, 1, 1 + 3i, 1 - 3i, 5.\]

Write the general solution of this differential equation.