MAT 303 Assignment 2.
Hand in to the instructor in class on Friday, September 22.

**Problem 1.** Consider the differential equation

\[ y' = (x + y)^{\frac{1}{3}} - 1. \]  \hspace{1cm} (1)

Describe all pairs of numbers \((x_0, y_0)\) for which Theorem of Existence and Uniqueness guaranties that the initial value problem \(y(x_0) = y_0\) has a unique solution.

**Problem 2.** Solve the differential equation (1). Describe all pairs \((x_0, y_0)\) for which the initial value problem \(y(x_0) = y_0:\)

a) has a unique solution,

b) do not have any solutions,

c) has more than one solution.

**Problem 3.** Separate variables and use partial fractions to solve the initial value problem

\[ \frac{dx}{dt} = 3x(5 - x), \quad x(0) = 8. \]

**Problem 4.** A tank contains 1000 liters \((L)\) of a solution consisting of 100 kg of salt dissolved in water. Pure water is pumped into the tank at the rate of 5\(L/s\), and the mixture – kept uniform by stirring – is pumped out at the same rate. How long will it be until only 10 kg of salt remains in the tank?

**Problem 5.** Verify that the given differential equation is exact; then solve it.

\[ \frac{1}{x} \sin y \, dx + (\ln x \cos y + y)dy = 0. \]
Problem 6. Show that the following differential equation is homogeneous:

\[ x(\ln x - \ln t + 1)dt = tdx. \]

Solve the initial value problem \( x(1) = 1 \).

Problem 7. The time rate of change of a rabbit population \( P \) is proportional to the square root of \( P \). At time \( t = 0 \) (months) the population numbers 100 rabbits and is increasing at the rate of 20 rabbits per month. How many rabbits will there be one year later?