1. The following table shows the preference schedule for an election with four candidates (A, B, C and D). Use the Borda method to find the complete ranking of the candidates.

<table>
<thead>
<tr>
<th>Number of voters</th>
<th>7</th>
<th>8</th>
<th>4</th>
<th>3</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>B</td>
<td>A</td>
<td>A</td>
<td>D</td>
<td>B</td>
</tr>
<tr>
<td>2nd</td>
<td>C</td>
<td>B</td>
<td>B</td>
<td>C</td>
<td>A</td>
</tr>
<tr>
<td>3rd</td>
<td>A</td>
<td>D</td>
<td>C</td>
<td>B</td>
<td>D</td>
</tr>
<tr>
<td>4th</td>
<td>D</td>
<td>C</td>
<td>D</td>
<td>A</td>
<td>C</td>
</tr>
</tbody>
</table>

Solution:

<table>
<thead>
<tr>
<th></th>
<th>7</th>
<th>8</th>
<th>4</th>
<th>3</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st (4)</td>
<td>B</td>
<td>A</td>
<td>A</td>
<td>D</td>
<td>B</td>
</tr>
<tr>
<td>2nd (3)</td>
<td>C</td>
<td>B</td>
<td>B</td>
<td>C</td>
<td>A</td>
</tr>
<tr>
<td>3rd (2)</td>
<td>A</td>
<td>D</td>
<td>C</td>
<td>B</td>
<td>D</td>
</tr>
<tr>
<td>4th (1)</td>
<td>D</td>
<td>C</td>
<td>D</td>
<td>A</td>
<td>C</td>
</tr>
</tbody>
</table>

A: 14 + 32 + 16 + 3 + 3 = 68
B: 28 + 24 + 12 + 6 + 4 = 74
C: 21 + 8 + 8 + 9 + 1 = 47
D: 7 + 16 + 4 + 12 + 2 = 41

1st B, 2nd A, 3rd C, 4th D
Answer: B, A, C, D
2. Find the complete ranking of the candidates from the election of problem 1 using the Plurality method. Explain using problem 1 that the Borda method violates the Majority fairness criterion.

**Solution:** Number of first place votes
- \( A: 8 + 4 = 12 \), \( B: 7 + 1 = 8 \),
- \( C: 0 \), \( D: 3 \)

1st A, 2nd B, 3rd D, 4th C

**Answer:** A, B, D, C.

Notice, there were \( 7 + 8 + 4 + 3 + 1 = 23 \) voters. A has \( 12 > \frac{23}{2} \) first place votes (majority). Majority criterion says A should win. However, in Borda count B is the winner. This violates the Majority criterion.
3. Find the Shapley-Shubik power indexes of the weighted voting system $[8 : 7, 5, 2]$. You can leave the answer in the form of a simple fraction (like $\frac{2}{7}$).

Solution

List all sequential coalitions, underline pivotal players.

$<P_1, P_2, P_3>$
$<P_1, P_3, P_2>$
$<P_2, P_1, P_3>$
$<P_2, P_3, P_1>$
$<P_3, P_1, P_2>$
$<P_3, P_2, P_1>$

Pivotal counts: $S_1 = 4$, $S_2 = S_3 = 1$

Indexes: $b_1 = \frac{4}{6} = \frac{2}{3}$, $b_2 = b_3 = \frac{1}{6}$

Answer: $b_1 = \frac{2}{3}$, $b_2 = b_3 = \frac{1}{6}$
4. A friend treated Layla and Steve with the half meatball - half vegetarian foot-long sandwich for $9. They plan to divide it using the divider-chooser method. Layla likes the meatball part three times more than the vegetarian, Steve likes meatball part two times more than the vegetarian. Layla divides the sandwich by one vertical cut and then Steve chooses the part he likes more. Describe the outcome (where does Layla makes the cut and which part Steve chooses) and give the value of the shares to Layla and Steve.

\[ \text{Solution} \quad \text{To get the division find out the values of parts for Layla.} \]

If \$x \text{ is the value of veg. part for her then } 3x \text{ is for meat. In total } x + 3x = 4x = 9. \text{ Thus, } x = \frac{9}{4} = 2.25. \text{ She needs to cut into equal value } \frac{9}{2} = \$4.5 = 2x \text{ parts. This is } \frac{2}{3} \text{ of the meat value, so the cut is at } \frac{2}{3} \cdot 6 = \frac{12}{3} = 4 \text{ inch} \]
To find out what Steve chooses calculate the values of parts for him.

\[ 000000 \]

\[ 0 \quad a \quad 6 \quad 12 \]

\[ 2y \quad \_ \quad y \]

If \$y\ for\ veg.\ then\ \$2y\ for\ meat

\[ y + 2y = 3y = \$9.\ Thus,\ y = 3, 2y = 6.\ The\ \frac{2}{3}\ of\ the\ meat\ ball\ part\ is\ worth\ \frac{2}{3} \cdot 6 = \$4\ for\ him,\ the\ rest's\ worth\ 9 - 4 = \$5.\]

Therefore, Steve chooses the part containing veg. The rest goes to Layla. It's worth \$5 to her.

Multiple choice answers:
1. a, 2. c, 3. c, 4. a