ERRATUM TO: "UNIQUE CONTINUATION RESULTS FOR RICCI CURVATURE AND APPLICATIONS"

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Abstract. Corrections are given to some of the proofs of the paper above

In this note, we point out and correct some errors in the proofs of the main results in the paper [1]. The main results themselves are correct as stated, but the proofs need modification.

To begin, the proof of Lemma 3.3 in [1] is incorrect. Thus, h in $[1,(3.12)]$ is the linearization of a mapping $\partial C \to \partial C$, and so has n degrees of freedom, not $(n+1)$ as indicated there. Moreover, the information given on solving the PDE in (3.12) is insufficient. In addition, the proof of Lemma 3.2 also requires the Nash-Moser implicit function theorem; the linearization L of $\mathcal{H}$ is not surjective since one has no gain of regularity in the τ direction.

In Lemma 1 below, we state a slightly more general version of Lemmas 3.2 and 3.3 of [1] and then proceed with the proof. We recall that the main point of these results is to construct a foliation of prescribed mean curvature with harmonic coordinates along the leaves, such that the lapse and shift are prescribed at the boundary. The rest of the work in [1, §3] then proceeds as before.

We begin by describing the initial set-up of the issue. Let C_0 be the unit ball $B^n(1)$ in Cartesian coordinates x^i, $1 \leq i \leq n$ and let $D_0 = C_0 \times [0,1]$ be the vertical cylinder over C_0 in coordinates $x^\alpha = (\tau, x^i)$. Let $\rho^2 = \sum (x^i)^2 - 1$ be the Euclidean distance (squared) to ∂C_0 and view the graph of the function $\tau = a^{-1}\rho$ as a cone with boundary ∂C_0. Let D_a be the interior solid cone, where $\tau \geq a^{-1}\rho$. We will assume $a >> 1$, so that the cone is almost flat.

Next consider mappings ϕ from (a neighborhood of) D_a, with range into the space with coordinates (t, y^i), which are diffeomorphisms onto their image, which are close to the identity, and for which $\phi = id$ on C_0. The horizontal level surfaces $\Sigma_\tau = \tau \times B^n$ of τ in D_a are mapped under ϕ to a foliation S_τ of the image domain. Given a metric g, close to the standard flat metric in the (t, y^i) coordinates, the pullback $\phi^* g$ is defined on D_a. We will only consider metrics which equal a given fixed metric g_0 on C_0, for which the coordinates y^i are harmonic on (C_0, g_0).

We seek foliations such that x^i are harmonic on each leaf Σ_τ with respect to $\phi^* g$, i.e.

$$\Delta^{\phi^* g}_{\Sigma_\tau} x^i = 0,$$

and such that

$$H^{\phi^* g}_{\Sigma_\tau}(\tau, x^i) = \psi(x^i),$$

where ψ is a given function, close to 0. In particular, this implies

$$\partial_\tau H^{\phi^* g}_{\Sigma_\tau} = 0.$$

The unit normal vector of the foliation Σ_τ in the metric $\phi^* g$ is given by $N_{\phi^* g} = u^{-1}(\partial_\tau - \sigma)$ where (u, σ) are the lapse and shift of the Σ_τ foliation. As in [1], one has $(\phi^* g)^{00} = u^{-2}$, $(\phi^* g)^{0i} = -u^{-2}\sigma^i$. Prescribing the lapse and shift of the τ-foliation Σ_τ in the (τ, x^i) coordinates at the boundary ∂D_a

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is thus equivalent to prescribing $(\phi^* g)^{0\alpha}$ on ∂D_a, or equivalently the unit normal vector $N_{\phi^* g}$ in (τ, x^i) coordinates.

Lemma 0.1. *Given a $C^{k-1,\alpha}$ metric g on D_a near the Euclidean metric, $k \geq 6$, $\psi \in C^{k-3,\alpha}(D_a)$ close to 0 and parameter a sufficiently large, there exists $\phi \in C^{k,\alpha}$ near the identity such that*

$$H^{\phi^* g}_{\Sigma_\tau}(\tau, x^i) = \psi(x^i), \tag{1}$$

$$\Delta^{\phi^* g}_{\Sigma_\tau} x^i = 0, \tag{2}$$

and such that

$$(N_{\phi^* g})|_{\partial D_a} = N, \tag{3}$$

for any given vector field $N \in C^{k-2,\alpha}$ close to the standard normal $(1, 0)$.

Proof: The proof is similar to the proof of this result in Lemmas 3.2 and 3.3 of [1], but differs in some of the details.

Let $X_k = Met_0^{k-1,\alpha}(\hat{D}_a)$, be the space of metrics which agree with a fixed metric g_0 on C_0 as above, where $\hat{D}_a$ is a fixed neighborhood of D_a in (t, y^i) space, (so $\phi = id$). Let $Y_k = \mathrm{Diff}_0^{k,\alpha}(D_a)$ be the space of the diffeomorphisms ϕ mapping D_a into (t, y^i) space and equal to the identity on the bottom leaf C_0. Finally let $Z_k = C^{k-3,\alpha}(D_a) \times \prod_1^n C_0^{k-3,\alpha}(D_a) \times \chi_0^{k-2,\alpha}(\partial D_a)$, where χ_0 is the space of vector fields at ∂D_a, (not necessarily tangent to ∂D_a), and vanishing on $C_0 \cap \partial D_a$. We will only consider small neighborhoods of the standard configuration in these spaces.

Consider then the map

$$\mathcal{H} : X_k \times Y_k \to Z_k$$

$$\mathcal{H}(g, \phi) = (H^{\phi^* g}_{\Sigma_\tau}(\tau, x^i) - \psi(x^i), \;\; \Delta^{\phi^* g}_{\Sigma_\tau} x^i, \;\; (N_{\phi^* g})|_{\partial D_a}). \tag{4}$$

This map actually maps into Z_{k+1}, in that the first two terms on the right in (4) are in $C^{k-2,\alpha}$ while the last term is in $C^{k-1,\alpha}$. However, the map $\mathcal{H}$, although continuous, is not smooth as a map into Z_{k+1}. This comes from the well-known fact that the group $\mathrm{Diff}^{k,\alpha}$ of $C^{k,\alpha}$ diffeomorphisms acts continuously on $Met^{k-1,\alpha}$ but not smoothly: if W is a $C^{k,\alpha}$ smooth vector field and $g \in Met^{k-1,\alpha}$ then $\mathcal{L}_W g$ is only $C^{k-2,\alpha}$ smooth. Thus we view $\mathcal{H}$ mapping into Z_k, and as such, it is then a smooth map of Banach manifolds.

To compute the linearization of $\mathcal{H}$ at (g, ϕ) in the 2nd or Y variable ϕ, fix g and let ϕ_s be a curve of diffeomorphisms with $\phi_s|_{s=0} = \phi$. Writing $\phi_s = \psi_s \circ \phi$, so $\phi_0 = id$, the derivative $v = \frac{d}{ds}\psi_s|_{s=0}$ is a vector field on D_a. Throughout the following, we work in the pull-back domain with (τ, x^i) coordinates. Let then $v = (v^0, v^i)$ with respect to the coordinates (τ, x^i).

We use the following standard formulas to compute the linearization in the ϕ-variable:

$$2H'_h = tr[\nabla_{\bar{N}} h - 2\delta^*(h(\bar{N})^T) - \delta^*(h_{00}\bar{N})], \tag{5}$$

$$\Delta'_h u = -\langle D^2 u, h\rangle + \langle du, \beta(h)\rangle,$$

where we have set $\bar{N} = N_{\phi^* g}$. Here β is the Bianchi operator on Σ_τ, and $h = 2\delta^* v$. Note that $h \in C^{k-2,\alpha}$, so that $H'_h \in C^{k-3,\alpha}$, as is $\Delta'_h u$, since $\beta(h) \in C^{k-3,\alpha}$.

To compute the first term in (5), write $v = \eta\bar{N} + v^T$ where v^T is tangent to Σ_τ. Then $2H'_{\delta^* v^T} = v^T(H)$. Also $\delta^*(\eta\bar{N}) = \eta A + d\eta \cdot \bar{N}$, where A is the 2nd fundamental form, so that $h(\bar{N})^T = d\eta^T$. Further straightforward calculation then shows that $2H'_{\delta^* v} = -\Delta_{\Sigma_\tau}\langle v, \bar{N}\rangle + v(H)$ while for the second term, again a standard calculation gives $\Delta'_{\phi_s^* g} x^i = -\Delta v^i$ + lower order terms.

Let then

$$L(v) = L_{g,\phi}(v) = (-\Delta_{\Sigma_\tau}\langle v, \bar{N}\rangle + v(H), -\Delta_{\Sigma_\tau} v^i + \ell^i(v^j, \partial v^j)). \tag{6}$$

where Δ_{Σ_τ} is the Laplacian and H the mean curvature of Σ_τ, both with respect to $\phi^* g$. The term ℓ^i is a lower order term, depending only on $\{v^j\}$ and its first derivatives, (tangent to Σ_τ). This is the interior operator, giving the first two components of the derivative $D_2\mathcal{H}$ in (4).

Given any $f = (f^0, \cdots, f^n) \in C^{k-3,\alpha}(\Sigma_\tau)$, there exists $v \in C^{k-1,\alpha}(\Sigma_\tau)$ such that

$$L(v) = f. \tag{7}$$

Thus, the interior operator is surjective, modulo loss of derivatives. More precisely, there is a loss of one derivative due to the lack of smoothness for the action of diffeomorphisms on metrics, and a loss of two derivatives since the operator L is elliptic only in the leaf directions Σ_τ, and not in τ. Since there is no boost in the regularity in the τ-direction, for $f \in C^{k-3,\alpha}(D_a)$, one has $v \in C^{k-3,\alpha}(D_a)$.

Of course there are many solutions of (7), and we wish to view the linearization of the last (boundary) term in (4) as boundary values for L.

To compute the linearization of $N_{\phi^* g}$ with respect to ϕ, fix g and let $\phi_s = \phi + sv + O(s^2)$ be a curve of diffeomorphisms, so that $v = d\psi_s/ds$ as above. We evaluate v at ∂D_a, but v is not necessarily tangent to ∂D_a. Then

$$\frac{d}{ds}\bar{N}_s = \frac{d}{ds}N_{\phi_s^* g} = -[\bar{N}, v] - \nabla^\Sigma \langle v, \bar{N}\rangle, \tag{8}$$

where ∇^Σ is the gradient tangent to the leaves Σ_τ. To verify this, if $h = 2\delta^* v$ is a variation of the metric $\phi^* g$, then $\bar{N}'_h = h(\bar{N})^\Sigma - \frac{1}{2}h_{00}\bar{N}$, where the superscript Σ is the tangential projection onto the leaves Σ_τ. Now $2(\delta^* v)(\bar{N}) = \nabla_{\bar{N}} v + \langle \nabla . v, \bar{N}\rangle = [\bar{N}, v] + \nabla_v \bar{N} + \langle \nabla . v, \bar{N}\rangle$. Working tangentially, write the last term here as $\nabla^\Sigma \langle v, \bar{N}\rangle - A(v) = \nabla^\Sigma\langle v, \bar{N}\rangle - \nabla_v \bar{N}$. This verifies (8) tangentially, and since $2\bar{N}\langle v, \bar{N}\rangle = h_{00}$, this verifies it also in the normal direction.

Thus, the linear operator we need to examine is

$$L(v) = (-\Delta_{\Sigma_\tau}\langle v, \bar{N}\rangle + v(H), -\Delta_{\Sigma_\tau} v^i + \ell^i(v^\alpha, \partial v^\alpha)), \tag{9}$$

with boundary condition

$$B(v) = -[\bar{N}, v] - \nabla^{\Sigma_\tau}\langle v, \bar{N}\rangle. \tag{10}$$

Note that for $v \in C^{k,\alpha}(\Sigma_\tau)$, $B(v) \in C^{k-2,\alpha}(\partial\Sigma_\tau)$.

The model case is the Euclidean cone D_a with slope a^{-1} and $\phi = id$. In this case, the operator is the Laplace operator

$$L_{\mathrm{eucl}}(v) = (-\Delta_{\Sigma_\tau} v^0, -\Delta_{\Sigma_\tau} v^i)$$

where $v^0 = \langle v, \bar{N}\rangle = \langle v, \partial_\tau\rangle$, and the boundary operator is

$$B_{\mathrm{eucl}}(v) = -\partial_\tau v - \nabla^{\Sigma_\tau} v^0.$$

One may now use cone-adapted coordinates

$$u = \tau, \quad r = \rho - a\,\tau,$$

angular coordinates being kept unchanged. Thus $\partial_\tau = \partial_u - a\partial_r$ and $\partial_\rho = \partial_r$, so that

$$B_{\mathrm{eucl}}(v) = \partial_u \begin{pmatrix} v^0 \\ v^\rho \\ v^\mu \end{pmatrix} - a \begin{pmatrix} \partial_r v^0 \\ \partial_r v^\rho \\ \partial_r v^\mu \end{pmatrix} + \begin{pmatrix} 0 \\ \partial_r v^0 \\ \partial_\mu v^0 \end{pmatrix}. \tag{11}$$

(Note here that we do not change coordinates in the unknown v.) The general boundary operator $B = B_{g,\phi}$ is a perturbation of B_{eucl}:

$$B = (1 - \epsilon_1(g,\phi))B_{\mathrm{eucl}} + \phi_1(g,\phi) + \phi_0(g,\phi), \tag{12}$$

where ϵ_j $(j = 1)$ is a function and ϕ_j $(j = 0, 1)$ are linear differential operators of order j, depending smoothly on the $(1-j)$ jet of $g - g_{\mathrm{eucl}}$ and $\phi - id$, and vanishing at $g = g_{\mathrm{eucl}}$, $\phi = id$. (Note that ϵ_1 involves in general some normal derivative ∂_r.)

We proceed as follows to solve the coupled system

$$L(v) = f, \qquad B(v) = z. \tag{13}$$

First, it is standard that it suffices to establish unique solvability of the homogeneous problem

$$L(v) = 0, \qquad B(v) = z. \tag{14}$$

Next, given any τ and $v_\tau \in C^{k-1,\alpha}(\partial\Sigma_\tau)$, there exists a unique extension, also called $v_\tau \in C^{k-1,\alpha}(\partial\Sigma_\tau)$, such that $L(v_\tau) = 0$, (given as the solution to the Dirichlet problem for L along Σ_τ. Let A denote the Dirichlet-to-Neumann map for this operator, so $A(v_\tau) = \partial_r v_\tau : C^{k-1,\alpha}(\partial\Sigma_\tau) \to C^{k-2,\alpha}(\partial\Sigma_\tau)$. Replacing the derivatives ∂_r in (12) by A, the equation (14) is then solved with initial condition $v|_{\tau=0} = 0$, provided the function $v_\tau = v(\tau, \cdot)$ satisfies the evolution equation

$$\partial_t v + (A + B_{\varepsilon,a}(t))\, v = z, \quad v(0, \cdot) = 0, \tag{15}$$

along ∂D_a. Here we use $t = au$ as a new coordinate along ∂D_a and $B_{\varepsilon,a}$ is a first-order perturbation continuous in $\varepsilon = |g - g_{\mathrm{eucl}}|_{C^{k-1,\alpha}} + |\phi - id|_{C^{k,\alpha}}$ and a^{-1}. (Since $a >> 1$, the last term in (12), when divided by a, may be treated as a perturbation as well.)

We solve the evolution problem (15) by seeking a generator for a continuous 1-parameter semigroup, as in the Hille-Yosida theorem, cf. [6]. Note here that the coefficients of the operator are t dependent. This is done by showing that the results of Tanabe [4],[5] and Kato [3], (giving certain generalizations of the Hille-Yosida theorem), apply to the situation at hand.

To begin, the Dirichlet-to-Neumann operator A is a nonnegative unbounded operator acting on L^2 of ∂D_a, and it is easily shown that, for a fixed $\lambda > 0$, large enough a, and small enough t and ε, $B_{\varepsilon,a}(t)(\lambda I + A)^{-1}$ is a continuous (bounded) operator on L^2 or any $C^{\ell,\alpha}$ of the boundary $\partial\Sigma_t$; the relevant case here is $\ell = k - 2$. Moreover, it converges in the norm topology to 0 as ε tends to 0. In fact, observe that

$$B_{\varepsilon,a}(t)(\lambda I + A)^{-1} = B_{\varepsilon,a}(t)(\lambda_0 I + A)^{-1} \left((\lambda - \lambda_0)(\lambda_0 I + A)^{-1} + I\right)^{-1},$$

which provides a uniform convergence for $\lambda > \lambda_0$.

We may then conclude that for $\varepsilon << 1$, $t << 1$, and a large enough, $\lambda I + A + B_{\varepsilon,a}(t)$ is invertible on the Hölder spaces $C^{\ell,\alpha}(\partial\Sigma)$, ($\ell \le k - 2$), for $\lambda > \lambda_0$. Moreover

$$|(\lambda I + A + B_{\varepsilon,a}(t))^{-1}| \le \frac{M}{\lambda}, \quad \text{for } \lambda > \lambda_0,$$

and this also extends to complex λ outside the half-line $\mathrm{Re}\,\lambda > \lambda_0$.

It is also obvious that, for $\lambda > \lambda_0$, $(\lambda I + A + B_{\varepsilon,a}(t))(\lambda I + A + B_{\varepsilon,a}(s))^{-1}$ is bounded independently of s, t in any small interval $[0, T]$. Moreover, still for $\lambda > \lambda_0$ and s, t in $[0, T]$,

$$|(\lambda I + A + B_{\varepsilon,a}(t))(\lambda I + A + B_{\varepsilon,a}(s))^{-1} - I| \le C|t - s|.$$

To prove the last inequality one needs to estimate the $C^{\ell,\alpha}(\partial\Sigma)$-norm, ($\ell = k - 2$), of $v_t - v_s$ for a given v_s, where v_t is defined through the following procedure: one defines first $w_s \in C^{\ell+1,\alpha}(\Sigma)$ such that

$$L(w_s) = \Delta_s w_s + p_s(w_s) = 0 \text{ in } \Sigma, \quad (\lambda I + A + B_{\varepsilon,a}(s))w_s = v_s \text{ on } \partial\Sigma,$$

($\Sigma = \Sigma_s$), where here A and B denote usual derivatives and not Dirichlet-to-Neumann-type operators. Then one may solve

$$L_t(w_t) = \Delta_t w_t + p_t(w_t) = 0 \text{ in } \Sigma, \quad w_t = v_s \text{ on } \partial\Sigma$$

and define $v_t = (\lambda I + A + B_{\varepsilon,a}(t))w_t$. Thus

$$|v_t - v_s|_{C^{\ell,\alpha}(\partial\Sigma)} \le C|w_t - w_s|_{C^{\ell+1,\alpha}(\Sigma)}.$$

But $w = w_t - w_s$ is a solution of

$$\Delta_s w + p_s(w) = (\Delta_t - \Delta_s)(w_s) + (p_t - p_s)(w_s) \text{ in } \Sigma, \quad w = 0 \text{ on } \partial\Sigma.$$

Hence

$$\begin{aligned}|w|_{C^{\ell+1,\alpha}(\Sigma)} &\le C|(\Delta_t-\Delta_s)(w_s)+(p_t-p_s)(w_s)|_{C^{\ell-1,\alpha}(\Sigma)}\\ &\le C|g_t-g_s|_{C^1}|w_s|_{C^{\ell+1,\alpha}(\Sigma)}\\ &\le C|t-s||w_s|_{C^{\ell+1,\alpha}(\Sigma)}\\ &\le C|t-s||v_s|_{C^{\ell,\alpha}(\partial\Sigma)},\end{aligned}$$

where the last line comes from the definition of w_s and the invertibility of the operator $\lambda I + A + B_{\varepsilon,a}(s)$ for $\lambda > \lambda_0$ and ϵ and a^{-1} small).

We can now apply Tanabe's result, [4], [5], see also Kato [3], (with $m = 1$ in condition (ii) of Theorem 1 of [3]) which shows existence and uniqueness in $[0, T]$ (for some small $T > 0$) of a solution of (15) with z in $C^{k-2,\alpha}$ of the boundary of the cone.

More precisely, the results of Kato and Tanabe yield a unique evolution operator $U(s, t)$ for $0 \le s \le t \le T$ which is continuous as a function of $[0, T]^2$ to the bounded linear maps of $C^{k-2,\alpha}(\partial\Sigma)$ in the strong topology. Moreover, it is continuously differentiable (still in the strong topology) and the range of $U(s, t)$ is a subspace of the domain $C^{k-1,\alpha}(\partial\Sigma_t)$ of $A + B_{\varepsilon,a}(t)$ for $s < t$. (Note here that the Tanabe-Kato result is first applied to the operator $A + B_{\varepsilon,a}(t) + 2\lambda_0$, from which deduces trivially the existence of an evolution operator for $A + B_{\varepsilon,a}(t)$). Thus, the solution to (15) may be written in the general form

$$v(t,\cdot) = \int_0^t U(t,s)z(s,\cdot)ds \tag{16}$$

and

$$v(t,\cdot) \in C^0([0,T], C^{k-2,\alpha}(\partial\Sigma)) \cap C^1((0,T], C^{k-2,\alpha}(\partial\Sigma)).$$

Moreover, each $v(t,\cdot)$ for $t > 0$ belongs to the domain $C^{k-1,\alpha}(\partial\Sigma)$ of $A + B_{\varepsilon,a}(t)$. More regularity can be obtained with standard methods (see e.g. [6], Chapters 40-42): from the equation

$$\partial_t v(t,\cdot) + (A + B_{\varepsilon,a}(t))v(t,\cdot) = z(t,\cdot) \in C^{k-2,\alpha}([0,T]\times\partial\Sigma), \tag{17}$$

and the fact that $v(0,\cdot) = 0$ is in the domain of $A + B_{\varepsilon,a}(t)$, one sees that $\partial_t v$ converges as $t \to 0$ in $C^{k-3,\alpha}(\partial\Sigma)$). Differentiating (17) in the t-variable and applying the Tanabe-Kato results to the solution $\partial_t v$ of

$$\partial_t(\partial_t v(t,\cdot)) + (A + B_{\varepsilon,a}(t))(\partial_t v(t,\cdot)) = \partial_t z(t,\cdot) - [\partial_t, A + B_{\varepsilon,a}(t)]v(t,\cdot),$$

$$\partial_t v(t,\cdot) \in C^0([0,T], C^{k-3,\alpha}(\partial\Sigma)) \cap C^1((0,T], C^{k-3,\alpha}(\partial\Sigma)),$$

with the same extra conclusions as above. By induction, this yields that

$$(\partial_t)^\ell v(t,\cdot) \in C^0([0,T], C^{k-2-\ell,\alpha}(\partial\Sigma)) \cap C^1((0,T], C^{k-2-\ell,\alpha}(\partial\Sigma),$$

and so in particular

$$v \in C^{k-3,\alpha}([0,T]\times\partial\Sigma).$$

This gives the existence of a right inverse operator

$$R : Z_k \to Y_{k-3}, \tag{18}$$

for the boundary value problem (13), so that $L \circ R = id$.

The rest of the proof is then essentially the same as that in [6]; namely one applies the Nash-Moser implicit function theorem, (in the version given by Zehnder). The verification of the hypotheses of Zehnder's theorem is straightforward and since there is a loss of three derivatives in (18), one obtains the statement in the Lemma with $k \ge 6$. (We expect that this regularity can be improved, but do not pursue this further here).

■

Next, we point out an error in the proof of Theorem 1.3 of [1]. In the setting and notation of [1], we applied Proposition 5.4 to the equation (5.45) with the choice of variation

$$h = (\delta^*(\frac{X}{t^n}))^T, \tag{19}$$

where $\delta^* = \delta^*_g$, so that $h = (t^{-n}\kappa)^T = \widetilde{\kappa}^T$.

The problem is that to apply Proposition 5.4, one needs h to be an infinitesimal Einstein deformation; although h in (19) is infinitesimal Einstein to high order, it is not exactly so. In fact, one must take h to be the full deformation tensor of X and not just the tangential part, which gives

$$h = \delta^*(\frac{X}{t^n}) = t^{-n}\delta^* X - nt^{-(n+1)}\nabla t \cdot X = t^{-n}\delta^* X + nt^{-n}N \cdot X, \tag{20}$$

where $N = \nabla r$ and $N \cdot X = \frac{1}{2}(N \otimes X + X \otimes N)$. The tangential part of (20) equals (19) but the mixed term was absent from the computation in [1]. This term, in effect, invalidates the remaining part of the proof.

Overcoming this computational error requires a rather different approach from that given in [1]. Since it is not closely related to the content of [1], the proof, (due to the first author), is given instead in [2].

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