MAT 531 SPRING 16 HOMEWORK 9

Due Tuesday, Apr 12

1. Let \((e^1, e^2, e^3)\) be the standard dual basis for \((\mathbb{R}^3)^\ast\). Show that \(e^1 \otimes e^2 \otimes e^3\) is not equal to a sum of an alternating tensor and a symmetric tensor.

2. Define a 2-form \(\Omega\) on \(\mathbb{R}^3\) by

\[\Omega = xdy \wedge dz + ydz \wedge dx + zdx \wedge dy.\]

(a). Compute \(\Omega\) in spherical coordinates \((\rho, \varphi, \theta)\) where \((x, y, z) = (\rho \sin \varphi \cos \theta, \rho \sin \varphi \sin \theta, \rho \cos \varphi)\).

(b). Compute \(d\Omega\) in both Cartesian and spherical coordinates and check that both expressions give the same 3-form.

(c) Compute the restriction \(\Omega|_{S^2} = \iota^*\Omega\) using coordinates \((\varphi, \theta)\), (on their domain of validity). Here \(\iota : S^2 \to \mathbb{R}^3\) is the standard inclusion of the unit sphere \(S^2(1)\) into \(\mathbb{R}^3\).

(d). Show that \(\Omega|_{S^2}\) is nowhere zero.

3. Spivak, # 11, Chapter 7

4. Spivak, # 21, Chapter 7