MAT 310 FALL 09 HOMEWORK 8

Due Wednesday, November 11

1. Let $(V, \langle , \rangle)$ be an inner product space and suppose $T : V \to V$ is a linear map. Show that if λ is an eigenvalue of T, then $\bar{\lambda}$ is an eigenvalue of the adjoint operator T^*.

2. Let $(V, \langle , \rangle)$ be an inner product space and suppose $T : V \to V$ is self-adjoint. If U is an invariant subspace of T, (so $T(U) \subset U$), show that $U^\perp$ is also an invariant subspace of T, so $T(U^\perp) \subset U^\perp$.

3. Let v be a given vector in an inner product space and define the linear functional $\ell(w) = \langle w, v \rangle$, so $\ell : V \to \mathbb{F}$. Find the formula for the adjoint linear map $\ell^* : \mathbb{F} \to V$.

4. Consider the linear operator T on $\mathbb{C}^2$ given by $T(z_1, z_2) = (3iz_1 - 2z_2, 4z_1 + 2iz_2)$. Find the formula for $T^*(z_1, z_2)$.

5. Let $C_0^\infty[0, 1]$ be the vector space of C^∞ real-valued functions f on $[0, 1]$ such that $f(0) = f(1) = 0$. These are the functions which are differentiable to all orders (infinite order). Define

$$T : C_0^\infty[0, 1] \to C^\infty[0, 1]$$
$$T(f) = f',$$

so T is the derivative operator. Use the L^2 inner product:

$$\langle f, g \rangle = \int_0^1 f(t)g(t)dt.$$

Find the formula for the adjoint T^*. Is T self-adjoint?

6. Suppose $T : V \to V$, where V is an inner product space and T is self-adjoint. Prove that any two eigenvectors of T which have different eigenvalues are necessarily orthogonal vectors in V.

7. Using Proposition 6.46, prove that

$$dim range T = dim range T^*$$

for $T : V \to W$ a linear map of finite dimensional vector spaces. Explain why this implies that the row rank of an $m \times n$ matrix equals its column rank. (The row (column) rank is the dimension of the span of the row (column) vectors in the matrix).