MAT 307 FALL 2012 MIDTERM I EXAM

Name: **ID Number:**

Problem	1	2	3	4	5	6	Total
Score							

Directions: Each problem is 15 points; some problems are easier, some harder. Do all of your work on these pages and cross out any work that should be ignored. Use the backs of pages if needed, indicating where you've done this. Notes, books, calculators and discussions with others are not permitted.

1. Find the *parametric* equation of the plane in $\mathbb{R}^3$ through the 3 points $(1,0,0)$, $(2,1,1)$, $(1,2,3)$.

Find the *non-parametric* equation of the plane.

2. Consider the linear system

$$\begin{aligned} x + y + z &= 1 \\ x - y &= 1 \\ x + y &= 2 \end{aligned}$$

a. Write this system in matrix form $A \cdot \mathbf{x} = \mathbf{b}$.

b. Find the row-reduced form of this linear system

c. Find all solutions of the linear system and describe the solution set.

3.

a. Find all real numbers a for which the matrix $\begin{pmatrix} 1 & a & 1 \\ 1 & -1 & 0 \\ 2 & 0 & a \end{pmatrix}$ has an inverse and does not have an inverse.

b. Find all solutions of the linear system and describe the structure of the solution set:

$$\begin{aligned} x + y + z &= 1, \\ x - y &= -1, \\ 2x + z &= 2. \end{aligned}$$

c. Find all solutions of the linear system and describe the structure of the solution set:

$$\begin{aligned} x + y + z &= 1, \\ x - y &= 1, \\ 2x + z &= 2. \end{aligned}$$

4. The function

$$c(t) = (e^t, e^{-t}, t)$$

is a curve in $\mathbb{R}^3$.

a. Compute the velocity vector of $c(t)$ at any t and find the equation of the tangent line to c at $t = 1$.

b. At what time t is the curve $c(t)$ closest to the origin $(0, 0, 0)$ and what is that distance? (Hint: Consider $|c(t)|^2$ as a function of t).

5.

a. Sketch the level curves of the function

$$f(x, y) = (x - y)^2$$

in the plane $\mathbb{R}^2$. Label your curves by the values of f on them, and choose at least 4 values of f to get a good picture of the behavior of the level curves over the full domain of this function.

b. Sketch the graph of f in $\mathbb{R}^3$.

6. Consider the function

$$f(x, y) = x^2 y^3.$$

a. Find the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at any (x, y).

b. Find the equation of the tangent plane to the graph of f at the point $(1, 1)$.