MAT 342 Applied Complex Analysis
Final Exam Example
May 2016

1. (12 pts, 4 pts each)
   a) Define the notion complex differentiable.
   
   b) Define the principle branch of the logarithm.
   
   c) State Cauchy’s residue theorem.
2. (12 pts, 4 pts each)
   a) Find the multiplicative inverse of $3 + 4i$ and write the solution in rectangular form.
   b) Find all $z \in \mathbb{C}$ such that $z^2 = 4i$.
   c) Prove the triangle inequality: For all $z, w \in \mathbb{C}$, the inequality

\[ |z + w| \leq |z| + |w| \]

holds.
3. (10 pts) Find all $z \in \mathbb{C}$ such that

$$z^4 + z^3 + z^2 + z + 1 = 0.$$
4. (12 pts) Let $f$ be an entire function such that

$$f(z) = f(z + 1) = f(z + i)$$

for all $z \in \mathbb{C}$. Prove that $f$ is constant.
5. (10 pts) Let $p$ be a polynomial of degree $d_p$ and let $q$ be a polynomial of degree $d_q$ with $\max\{d_p, d_q\} \geq 1$. Assume that $q$ is not constantly 0 and that $p$ and $q$ do not share a common zero. Let $f : \mathbb{C} \setminus \{z \in \mathbb{C} \mid q(z) = 0\} \to \mathbb{C}$ be given by

$$f(z) = \frac{p(z)}{q(z)}.$$

Let $z_0 \in \mathbb{C}$. Prove that there exists some $z \in \mathbb{C}$ such that $f(z) = z_0$. 

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6. (12 pts) Find the Laurent series of

\[ f(z) = \frac{1}{(z-1)(z-3)} \]

in \( \{ z \in \mathbb{C} \mid 0 < |z - 1| < 2 \} \).
7. (12 pts, 4 pts each) Let
\[ f(z) = \frac{1}{(z - 2)(z - 4)}. \]
Find the contour integrals of \( f \) along the circles about the origin of radius 1, 3 and 5, taken in counterclockwise direction.
8. (20 pts, 10 pts each) Compute both

a) \[ \int_{0}^{\infty} \frac{1}{1 + x^4} \, dx \]

and

b) \[ \int_{-\infty}^{\infty} \frac{x \sin(ax)}{x^4 + 4} \, dx \quad \text{where} \quad a > 0 \]

using residues.
Name: ___________________________  ID: ______________
Name: ___________________________ ID: _______________