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Dear David,

I seem to remember that you wondered whether the representations in a single Langlands L-class are necessarily all unitary or all nonunitary. The answer is "no," and here is an example.

The example occurs in $SU(3,2)$ attached to a maximal parabolic. Here M is connected and is locally the product of a circle and $SU(2,1)$. With roots formed relative to the diagonal subalgebra, the roots of M are $\pm(e_1 - e_2)$, $\pm(e_2 - e_5)$, and $\pm(e_1 - e_5)$. Define two discrete series of M to have Harish-Chandra parameters

$$\lambda_0 = (e_1 - e_5) + \tfrac{1}{2}(e_3 + e_4)$$
$$\lambda_0' = (e_1 - e_2) + \tfrac{1}{2}(e_3 + e_4),$$

and regard $\mathfrak{a}$ as built from the Cayley transform $\underset{\sim}{c}(\alpha)$ of $\alpha = e_3 - e_4$. Proposition 9.1 (p. 35) of the paper "The role of basic cases ..." by Birgit and me says the complementary series for λ_0 extends to $\lambda_0 + \frac{3}{2}\underset{\sim}{c}(\alpha)$, whereas for λ_0' it extends only to $\lambda_0 + \frac{1}{2}\underset{\sim}{c}(\alpha)$. Then the (irreducible) induced representations corresponding to $\lambda_0 + \underset{\sim}{c}(\alpha)$ and $\lambda_0' + \underset{\sim}{c}(\alpha)$ are in the same L-class, the first is infinitesimally unitary, and the second is not infinitesimally unitary.

I'll give a copy of this letter to anyone who raises the same question to me.

Best,

Tony