October 19, 1982

Dear David,

I seem to remember that you wondered whether the representations in a single Langlands L-class are necessarily all unitary or all nonunitary. The answer is "no," and here is an example.

The example occurs in SU(3,2) attached to a maximal parabolic. Here M is connected and is locally the product of a circle and SU(2,1). With roots formed relative to the diagonal subalgebra, the roots of M are ±(e_1 - e_2), ±(e_2 - e_3), and ±(e_1 - e_3). Define two discrete series of M to have Harish-Chandra parameters

\[ \lambda_0 = (e_1 - e_3) + \frac{1}{2}(e_3 + e_4) \]
\[ \lambda'_0 = (e_1 - e_2) + \frac{1}{2}(e_3 + e_4), \]

and regard \( \alpha \) as built from the Cayley transform \( \zeta(\alpha) \) of \( \alpha = e_3 - e_4 \). Proposition 9.1 (p. 35) of the paper "The role of basic cases ..." by Birgit and me says the complementary series for \( \lambda_0 \) extends to \( \lambda_0 + \frac{3}{2}\zeta(\alpha) \), whereas for \( \lambda'_0 \) it extends only to \( \lambda_0 + \frac{1}{2}\zeta(\alpha) \). Then the (irreducible) induced representations corresponding to \( \lambda_0 + \zeta(\alpha) \) and \( \lambda'_0 + \zeta(\alpha) \) are in the same L-class, the first is infinitesimally unitary, and the second is not infinitesimally unitary.

I'll give a copy of this letter to anyone who raises the same question to me.

Best,

Tony