

PREFACE TO THE SECOND EDITION

The publication of a second edition is an opportunity to underscore that the subject of Lie groups is important both for its general theory and for its examples. To this end I have added material at both the beginning and the end of the first edition.

At the beginning is now an Introduction, directly developing some of the elementary theory just for matrix groups, so that the reader at once has a large stock of concrete and useful examples. In addition, the part of Chapter I summarizing the full elementary theory of Lie groups has been expanded to provide greater flexibility where one begins to study the subject. The goal has been to include a long enough summary of the elementary theory so that a reader can proceed in the subject confidently with or without prior knowledge of the detailed foundations of Lie theory.

At the end are two new chapters, IX and X. Partly these explore specific examples and carry the theory closer to some of its applications, especially infinite-dimensional representation theory. Chapter IX is largely about branching theorems, which have applications also to mathematical physics and which relate compact groups to the structure theory of noncompact groups. Chapter X is largely about actions of compact Lie groups on polynomial algebras. It points toward invariant theory and some routes to infinite-dimensional representation theory.

The reader's attention is drawn to the Historical Notes near the end of the book. These notes often put the content of the text in a wider perspective, they supply certain details that have been omitted in the text, and they try to anticipate and answer questions that the reader might ask.

Here is more detail about how the second edition differs from the first, apart from minor changes: The Introduction is all new, expanding upon two pages from §I.10 of the first edition. The main change within Chapter I is that the discussion of the elementary theory of Lie groups in §10 has been expanded into four sections, providing more detail about the theory itself and adding material about covering groups, complex structures and complex Lie groups, and the real analytic structure of Lie groups. Results about the largest nilpotent ideal in a Lie algebra have been added to §6, and the section on classical semisimple Lie groups has been adjusted to be

compatible with the new Introduction. In addition, some of the problems at the end of the chapter have been replaced or adjusted.

In Chapters II through VIII, the text contains only a few significant additions. A new Proposition 2.13 improves on the first edition's Corollary 2.13 by enabling one to recognize subalgebras of a complex semisimple Lie algebra that can be extended to Cartan subalgebras. To §III.3 has been added the left Noetherian property of universal enveloping algebras. A paragraph has been added at the beginning of §IV.3 to smooth the transition to the new Chapter IX. In Chapter VII, Sections 1 and 9 make use of new material from Chapter I concerning complex structures. In Chapter VIII, a misleading and incorrect example in §5 has been excised, and Lemma 8.57 represents a tightening of the proof of Theorem 8.60. Most chapters from II through VIII contain additional problems, either just before the blocks of related problems begin or at the very end. One block of problems in Chapter V has been postponed to Chapter IX.

Chapters IX and X are new to the second edition. Appendix A contains a new section on left Noetherian rings, and Appendix B contains new sections that state and prove Ado's Theorem and the Campbell–Baker–Hausdorff Formula. The Historical Notes and the References have been expanded to take the new material into account.

The only chapter in which sections have been renumbered is Chapter I, and the only places in which results have been renumbered are in Chapters I and III and in Appendix A.

In writing the second edition, I was greatly assisted by Paul Friedman, who read and criticized several drafts, spending a great deal of time helping to get the exposition right. I could not have finished the project successfully without him and am extremely grateful for his assistance. I was helped also by P. Batra, R. Donley, and D. Vogan, who told me of the errors that they had found in the first edition, and I thank them for their efforts.

Much of the second edition was prepared while I was a visitor at the Institute for Advanced Study, and I appreciate the Institute's hospitality. As was the case with the first edition, the typesetting was by $\mathcal{A}\mathcal{M}\mathcal{S}\text{-}\mathcal{T}\mathcal{E}\mathcal{X}$, and the figures were drawn with Mathematica®.

June 2002