PREFACE TO THE FIRST EDITION

Fifty years ago Claude Chevalley revolutionized Lie theory by publishing his classic *Theory of Lie Groups I*. Before his book Lie theory was a mixture of local and global results. As Chevalley put it, "This limitation was probably necessary as long as general topology was not yet sufficiently well elaborated to provide a solid base for a theory in the large. These days are now passed."

Indeed, they are passed because Chevalley's book changed matters. Chevalley made global Lie groups into the primary objects of study. In his third and fourth chapters he introduced the global notion of analytic subgroup, so that Lie subalgebras corresponded exactly to analytic subgroups. This correspondence is now taken as absolutely standard, and any introduction to general Lie groups has to have it at its core. Nowadays "local Lie groups" are a thing of the past; they arise only at one point in the development, and only until Chevalley's results have been stated and have eliminated the need for the local theory.

But where does the theory go from this point? Fifty years after Chevalley's book, there are clear topics: É. Cartan's completion of W. Killing's work on classifying complex semisimple Lie algebras, the treatment of finite-dimensional representations of complex semisimple Lie algebras and compact Lie groups by Cartan and H. Weyl, the structure theory begun by Cartan for real semisimple Lie algebras and Lie groups, and harmonic analysis in the setting of semisimple groups as begun by Cartan and Weyl.

Since the development of these topics, an infinite-dimensional representation theory that began with the work of Weyl, von Neumann, and Wigner has grown tremendously from contributions by Gelfand, Harish-Chandra, and many other people. In addition, the theory of Lie algebras has gone in new directions, and an extensive theory of algebraic groups has developed. All of these later advances build on the structure theory, representation theory, and analysis begun by Cartan and Weyl.

With one exception all books before this one that go beyond the level of an introduction to Lie theory stick to Lie algebras, or else go in the direction of algebraic groups, or else begin beyond the fundamental "Cartan decomposition" of real semisimple Lie algebras. The one exception is the book Helgason [1962],* with its later edition Helgason [1978]. Helgason's books follow Cartan's differential-geometry approach, developing geometry and Lie groups at the same time by geometric methods.

The present book uses Lie-theoretic methods to continue Lie theory beyond the introductory level, bridging the gap between the theory of complex semisimple Lie algebras and the theory of global real semisimple Lie groups and providing a solid foundation for representation theory. The goal is to understand Lie groups, and Lie algebras are regarded throughout as a tool for understanding Lie groups.

The flavor of the book is both algebraic and analytic. As I said in a preface written in 1984, "Beginning with Cartan and Weyl and lasting even beyond 1960, there was a continual argument among experts about whether the subject should be approached through analysis or through algebra. Some today still take one side or the other. It is clear from history, though, that it is best to use both analysis and algebra; insight comes from each." That statement remains true.

Examples play a key role in this subject. Experts tend to think extensively in terms of examples, using them as a guide to seeing where the theory is headed and to finding theorems. Thus examples properly play a key role in this book. A feature of the presentation is that the point of view—about examples and about the theory—has to evolve as the theory develops. At the beginning one may think about a Lie group of matrices and its Lie algebra in terms of matrix entries, or in terms of conditions on matrices. But soon it should no longer be necessary to work with the actual matrices. By the time one gets to Chapters VII and VIII, the point of view is completely different. One has a large stock of examples, but particular features of them are what stand out. These features may be properties of an underlying root system, or relationships among subgroups, or patterns among different groups, but they are far from properties of concrete matrices.

A reader who wants only a limited understanding of the examples and the evolving point of view can just read the text. But a better understanding comes from doing problems, and each chapter contains some in its last section. Some of these are really theorems, some are examples that show the degree to which hypotheses can be stretched, and some are exercises. Hints for solutions, and in many cases complete solutions, appear in a section near the end of the book. The theory in the text never relies on

*A name followed by a bracketed year points to the list of references at the end of the book.

a problem from an earlier chapter, and proofs of theorems in the text are never left as problems at the end of the current chapter.

A section called Historical Notes near the end of the book provides historical commentary, gives bibliographical citations, tells about additional results, and serves as a guide to further reading.

The main prerequisite for reading this book is a familiarity with elementary Lie theory, as in Chapter IV of Chevalley [1946] or other sources listed at the end of the Notes for Chapter I. This theory itself requires a modest amount of linear algebra and group theory, some point-set topology, the theory of covering spaces, the theory of smooth manifolds, and some easy facts about topological groups. Except in the case of the theory of involutive distributions, the treatments of this other material in many recent books are more consistent with the present book than is Chevalley's treatment. A little Lebesgue integration plays a role in Chapter IV. In addition, existence and uniqueness of Haar measure on compact Lie groups are needed for Chapter IV; one can take these results on faith or one can know them from differential geometry or from integration theory. Differential forms and more extensive integration theory are used in Chapter VIII. Occasionally some other isolated result from algebra or analysis is needed; references are given in such cases.

Individual chapters in the book usually depend on only some of the earlier chapters. Details of this dependence are given on page xvii.

My own introduction to this subject came from courses by B. Kostant and S. Helgason at M.I.T. in 1965–67, and parts of those courses have heavily influenced parts of the book. Most of the book is based on various courses I taught at Cornell University or SUNY Stony Brook between 1971 and 1995. I am indebted to R. Donley, J. J. Duistermaat, S. Greenleaf, S. Helgason, D. Vogan, and A. Weinstein for help with various aspects of the book and to the Institut Mittag-Leffler for its hospitality during the last period in which the book was written. The typesetting was by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX, and the figures were drawn with Mathematica®.

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