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T. N. Bailey
A. W. Knap
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Foreword

In 1997 the annual instructional conference of the International Centre for Mathematical Sciences in Edinburgh was devoted to the representation theory of semisimple groups, to automorphic forms, and to the relations between these subjects. It was organized by T. N. Bailey, L. Clozel, M. Duffo, and A. W. Knap. The two-week meeting began with a rapid summary of basic theory and concluded with two lectures by Robert Langlands, returning from the award of the Wolf Prize. In between, fifteen other world experts gave courses of two to five lectures. There were close to one hundred participants, largely from Western Europe and North America, but also from Eastern Europe, Japan, and the Developing World. Funding for the conference was provided by the European Commission and the Engineering and Physical Sciences Research Council of the United Kingdom.

The papers in this volume consist of slightly expanded versions of the lectures, with some minor rearrangements. An exception is the paper by James Arthur, which is a version of a lecture given at a later conference. All papers were received before May 1, 1997, and were refereed. The papers are intended to provide overviews of the topics they address, and the authors have supplied extensive bibliographies to guide the reader who wants more detail. The editors hope that the papers will serve partly as guides to the literature and that readers at any level will be able to get an outline of new ideas that they will be able to fill in by following the references. As is true in the mathematical literature generally, different authors use slightly different definitions and notation. A global index at the end of the volume may help the reader reconcile the differences.

The aim of the conference was to provide an intensive treatment of representation theory for two purposes: One was to help analysts to make systematic use of Lie groups in work on harmonic analysis, differential equations, and mathematical physics, and the other was to treat for number theorists the representation-theoretic input to Wiles's proof of Fermat's Last Theorem.

It is tempting to think of the lectures and papers as consisting of a common core and two more advanced parts—one going in the direction of analysis on semisimple groups G and semisimple symmetric spaces G/H and the other going in the direction of properties of cusp and automorphic forms, their associated number theory, and properties of G/Γ for arithmetic subgroups Γ . But the editors have resisted the temptation to organize the proceedings in this fashion, because this would ignore the important historical interplay between the two subjects.

This interplay goes in both directions, as evidenced in many of the papers. The Langlands conjecture on discrete series of G , which is discussed in Schmid's paper, came about when Langlands took a known theorem about G/Γ , put $\Gamma = 1$, and made a heuristic calculation about what should happen. The standard intertwining

operators for G , which are discussed in van den Ban's article, originally arose in the setting of G/Γ , but their beautiful properties are much clearer in the setting of G and lead to a better understanding of analytic continuation of Eisenstein series and L functions. Harish-Chandra's harmonic analysis on G , which is discussed in Helgason's paper, used Eisenstein integrals and cusp forms modeled on Eisenstein series and cusp forms for G/Γ . In turn Harish-Chandra's analysis on G is in part the model for analysis on the semisimple symmetric spaces G/H , discussed in the paper by van den Ban, Flensted-Jensen, and Schlichtkrull. Oddly, the analysis on G/H adapts two devices, truncation and the residual spectrum, that were first used for G/Γ but are not necessary in the analysis for G .

A great deal of the number-theoretic part of the representation theory in this volume is devoted to functoriality, a conjectural notion introduced by Langlands and applicable only in the setting of G/Γ . Rogawski's article shows how instances of functoriality lead to the Langlands proof of previously unsettled cases of Artin's conjecture; in turn, these cases of Artin's conjecture are what Wiles used from representation theory in his proof of Fermat's Last Theorem.

An important tool in addressing functoriality is the trace formula, which is discussed in several papers. One final instance of the interplay between G/Γ and G/H is that the notion of a semisimple symmetric space, which involves the fixed group of an involution, can be adapted from Lie groups to algebraic groups defined over number fields. In Jacquet's article this notion leads to a relative trace formula and to a conjecture characterizing the key ingredient, base change, in the work of Langlands on Artin's Conjecture. In his own article Langlands speculates that this formula of Jacquet is worth further examination by the coming generation.

The editors are grateful to David Vogan for his assistance with mathematical editing, to Lucy Young and Margaret Cook for making the arrangements for the conference, and to Sergei Gelfand, Christine Thivierge, and Thomas Costa at the American Mathematical Society for their work in publishing these proceedings.