Corrections to
Basic Real Analysis

Short Corrections

Page 4, last two lines of proof of Corollary 1.5. Replace with
" $\frac{n+1}{N} < y$. Thus $x < \frac{n+1}{N} < y$, and the rational number $r = \frac{n+1}{N}$ has the required properties."

Page 9, first display. Change "for only finitely many a" to
"for only finitely many n".

Page 12, line -7 of proof. Change " $0 \le k \le 2^{-n}$ " to " $0 \le k \le 2^n$ ".

Page 12, line -2 of proof. Change "$k \le 2^{-n}$ " to "$k \le 2^n$ " and "$p_{2^{-n}} = b$" to "$p_{2^n} = b$ ".

Page 16, three lines before second display. Change "parameter (j in our case)" to "parameter (i in our case)", and change "the variable i" to "the variable j".

Page 17, line 7. Change "$\sup_{x\in E}$" to "$\sup_{x\in S}$."

Page 18, lines 10–12. Change the two sentences
"By (b), choose $j_1 \ge j_0$ such that $|b_{ij} - L'_j| < 6\varepsilon$ whenever $i \in \{1, \ldots, i_1-1\}$ and $j \ge j_1$. Then $j \ge j_1$ implies $|b_{ij} - L'_j| < 6\varepsilon$ for all i whenever $j \ge j_1$ "
to
"By (ii), choose $i_2 \ge i_1$ such that $|b_{ij} - L'_j| < 6\varepsilon$ whenever $j \in \{1, \ldots, j_0-1\}$ and $i \ge i_2$. Then $i \ge i_2$ implies $|b_{ij} - L'_j| < 6\varepsilon$ for all j whenever $i \ge i_2$ ".

Page 20, line 8. Change " $\sum_{k=m+1}^n$" to " $\sum_{k=m}^n$".

Page 23, first display. Change "f" to "f_n " in three places.

Page 23, second paragraph, line 3. Change "$|x_k - y_k| < \delta_{n_k}(\epsilon)$" to "$|x_k - y_k| < 1/k$", and change "$|f_k(x_k) - f_k(y_k)| \ge \epsilon$" to "$|f_{n_k}(x_k) - f_{n_k}(y_k)| \ge \epsilon$".

Page 23, second paragraph, lines 10–11. Change "f_{k_l}" to "$f_{n_{k_l}}$" in 6 places.

Page 23, second paragraph, line 12. Change "f_k" to "f_{n_k}" in 2 places.

Page 23, line -4. Insert the following sentence after "uniformly Cauchy."
"Redefining our indices, we may assume that $n_k = k$ for all k".

Page 23, line -3. Change "exhibiting equicontinuity" to "exhibiting uniform equicontinuity".

Page 31, line 3 of Lemma 1.28. Change "$[a, b]-$" to $[a, b]\cap$".

Page 33, lines 12–16. Change "If $c \ge 0$, then … from Lemma 1.25e" to the following:
"If $c \ge 0$, then $M'_i = cM_i$ and $m'_i = cm_i$, so that $U(P, cf) = cU(P, f)$ and $L(P, cf) = cL(P, f)$. If $c \le 0$, then $M'_i = cm_i$ and $m'_i = cM_i$, so that $U(P, cf) = cL(P, f)$ and $L(P, cf) = cU(P, f)$. In either case, $U(P, cf) - L(P, cf) = |c|(U(P, f) - L(P, f))$, and (b) follows from Lemma 1.25e".

Page 34, line 4 of example. Change "$\ne 0 = \lim_n \int f_n\,dx$ " to
"$\ne 0 = \int \lim_n f_n\,dx$ ".

Page 37, lines -10 and -8. Change "ξ" to "ξ_i".

Page 37, lines -6 and -1. Change "$M_i M_i^* \Delta y_i$" to "$\sum M_i M_i^* \Delta y_i$".

Page 38, line -7. Change right side from "$2\epsilon M(B-A)$" to "$2\epsilon M(B-A)+\epsilon$".

Page 38, line -6. Change "$\epsilon + 3\epsilon M(B-A)$" to "$2\epsilon + 3\epsilon M(B-A)$".

Page 39, last four lines and top three lines of page 40. This material contained a subtle mathematics error. Beginning with "The sum of the terms of the second kind," change this material to read as follows:

"For the terms of the second kind, fix attention on one subinterval I^* of P^* and consider all the subintervals I_i of P that are of the second kind and lie within I^*. Let $|I_i|$ be the length of I_i, and let m_i be the supremum of f, positive or negative or zero, on I_i. Let m be the supremum of f on I^*. Then the contribution of the intervals I_i to $U(P,f)$ is $\sum_i m_i|I_i|$, and term by term this is $\leq \sum_i m|I_i| = m\sum_i |I_i|$. The intervals I_i must fill up I^* except possibly for a part of I^* at each end, and these two ends have a total length of $\leq \mu(P)$. Thus the contribution of the intervals I_i of the second kind inside I^* to $U(P,f)$ is

$$\begin{aligned}&\leq m\sum_i |I_i| = m(|I^*| - |\text{left end}| - |\text{right end}|)\\&\leq m|I^*| + |m|2\mu(P) \leq m|I^*| + (\sup|f|)2\mu(P) \leq m|I^*| + 2\epsilon/k.\end{aligned}$$

On the right side the term $m|I^*|$ is the term of $U(P^*,f)$ coming from I^*. Summing over the k intervals I^* whose union is $[a,b]$, we see that the contribution to $U(P,f)$ of all intervals of the second kind is

$$\leq U(P^*,f) + 2\epsilon.$$

Thus

$$U(P,f) \leq \epsilon + U(P^*,f) \leq \int_a^b f\,dx + 3\epsilon.$$

Similarly we can produce δ_2 such that $\mu(P) \leq \delta_2$ implies

$$L(P,f) \geq \int_a^b f\,dx - 3\epsilon.$$

If $\delta = \min\{\delta_1, \delta_2\}$ and $\mu(P) \leq \delta$, then

$$\int_a^b f\,dx - 3\epsilon \leq L(P,f) \leq S(P,f) \leq U(P,f) \leq \int_a^b f\,dx + 3\epsilon,$$

and hence $\left|S(P,f) - \int_a^b f\,dx\right| \leq 3\epsilon$."

Page 42, line 12. Change "$c = p + iq$" to $c = r + is$".

Page 48, line 3. Change "$\sum_{n=0}^{\infty}$" to "$\sum_{n=0}^{N}$".

Page 49, line -8. Change "$[0,+\infty)$" to "$[1,+\infty)$".

Page 51, line 9 of proof of Corollary 1.45. Change "exactly" to "at least" in order to allow for the fact that two of the complex numbers $i^n(x+iy)$ with $0 \le n \le 3$ have real and imaginary parts ≥ 0 if $(x,y)=(1,0)$ or $(x,y)=(0,1)$.

Page 51, line -6. Change "$(\sin x)/\cos x)$" to "$(\sin x)/(\cos x)$".

Page 52, line 2 of Example. Change "of f by" to "of F by".

Page 55, line 2 of Example. Change "$\log(1+t)$" to "$\log(1+x)$".

Page 56, line 1. Change "complex" to "numerical".

Page 56, line 3. Change "$\mathbb{C}$" to "$\mathbb{R}$".

Page 56, line 9 of Example. Change "$s_k = \sum_{k=1}^{\infty}$" to "$s_n = \sum_{k=1}^{n}$".

Page 58, line 4 of Section 9. Change "on a bounded interval" to "on a bounded interval $[a,b]$".

Page 59, line 2 of proof. Change "$\int_{-1/\sqrt{n}}^{1/\sqrt{n}}(1-x^2)\,dx$" to "$\int_{-1/\sqrt{n}}^{1/\sqrt{n}}(1-x^2)^n\,dx$".

Page 59, line 1 of caption of Figure 1.1. Change "$c_n\int_{-1}^{1}(1-x^2)^n\,dx$" to "$c_n(1-x^2)^n$" and delete one occurrence of the word "for".

Page 62, line 2, change "even" to "odd".

Page 62, line 3, change "odd" to "even".

Page 65, line -1. Change "$\sum_{n=1}^{N}(\sin nx)/n$" to "$\sum_{k=1}^{n}(\sin kx)/k$".

Page 70, line 5 of Theorem 1.57. Change "s_n" to "s_N".

Page 70, line -8. Change "$\int_{-\pi}^{\pi} D_N(x)\,dx = 1$" to "$\frac{1}{2\pi}\int_{-\pi}^{\pi} D_N(x)\,dx = 1$".

Page 72, condition (iii). Change "as n tends to infinity" to "as N tends to infinity".

Page 73, line -5 of proof of Theorem 1.59. Change "Thus a second use of (i) shows" to "Thus use of (i) and (ii) shows".

Page 75, line before Lemma 1.63. Change "$\frac{M'}{2\pi}$" to "M'".

Page 76, line -12. Change "$g(t) \ge f(t)$ implies $I \le g(t) \le f(t) \le S$" to "$g(t) \ge f(t)$ implies $I \le f(t) \le g(t) \le S$".

Page 76, line -9. Change "$|f+g|$" to $|f-g|$".

Page 76, line -8. Change "$\sup\limits_{t\in[x_{i-1},x_i]} |f(t)+g(t)|$" to "$\sup\limits_{t\in[x_{i-1},x_i]} |f(t)-g(t)|$".

Page 77, line -8. Change "$\sin x$" to "$\sin nx$", and change "$\cos x$" to "$\cos nx$".

Page 78, Problem 1. This problem needs an extra step so as to be manageable as the first problem in the book. Therefore change it to read as follows:

"(a) Derive the archimedean property (Corollary 1.3) from the convergence of bounded monotone increasing sequences (Corollary 1.6).

"(b) Using (a), derive the least-upper bound property (Theorem 1.1) from the convergence of bounded monotone sequences (Corollary 1.6)."

Page 79, Problem 12b. Change "$\int_1^x$" to "$\int_0^x$".

Page 79, Problem 13. Change "$0 < x < \pi$" to "$-1 < x < 1$", and change "$0 < x < \pi/2$" to "$-1 < x < 1$".

Page 79, Problem 19, line 4. Change "for all c" to "for all x".

Page 80, Problem 21, last line. Change "$g'(x) = 0$" to "$g'(0) = 0$".

Page 80, last line. Change "for $k \geq 0$" to "for $k \geq 1$".

Page 81, Problem 27, line 1. Change "$s_n(f; x)$" to "$s_n(f; \theta)$".

Page 84, line -1. Change "$b = y - z$" to "$b = z - y$".

Page 90, line 2 of proof of Lemma 2.2. Change "$0 \leq \big|x - \|y\|^{-2}(x, y)y\big|^2$" to "$0 \leq \big\|x - \|y\|^{-2}(x, y)y\big\|^2$".

Page 93, line 9. Change "**limit point** of X" to "**limit point** of A".

Page 96, line -2. Change "for fixed x is continuous in y" to "for fixed y is continuous in x".

Page 99, line 10. Change displayed line to read

$$|f_n(s)| \leq |f_n(s) - f(s)| + |f(s) - f_N(s)| + |f_N(s)| \leq 2 + |f_N(s)|$$

Page 99, line 11. Change "$+1$" to "$+2$".

Page 107, line 2. Change "over" to "open".

Page 107, line -4. Change "L^2 into L^1" to "$\mathcal{R}[-\pi, \pi]$ relative to L^2 into $\mathcal{R}[-\pi, \pi]$ relative to L^1".

Page 110, proof of Corollary 2.37, line 5. Change "convergent subsequence" to "subsequence convergent in $\mathbb{R}^n$. By Corollary 2.23 the limit is in C".

Page 112, line -12. Change "$|Q(z)| \leq$" to "$|Q(re^{i\theta})| \leq$".

Page 114, line -6 of proof of Theorem 2.46. Change "2^{-m}" to "$1/m$".

Page 114, line -5 of proof of Theorem 2.46. Change "2^{-m}" to "$1/m$".

Page 115, line -1. Change "$[c, u)$" to "$[c, x)$".

Page 116, line 1 of proof of Proposition 2.49. Change "be metric spaces" to "be metric spaces with X connected".

Page 119, line -3. Change "of a function" to "of a function on a metric space U".

Page 120, statement of Lemma 2.55. The hypothesis "complete" is not needed as long as completeness is not assumed in the definition of "oscillation". The metric space may therefore be taken to be (U, d) instead of (X, d).

Page 120, proof of Theorem 2.54. This proof is correct as written but can be simplified. Theorem 2.53 is needed only when invoked the second time, not the first. The simplification is to change the first three lines of the proof to read
"In view of Lemma 2.55 and the fact that U is open, it is enough to prove for each $\epsilon > 0$ that $\{x \in U \mid \operatorname{osc}_f(x) \geq \epsilon\}$ does not contain a nonempty open subset"
and to change on lines -8 and -7 the words "Again by Theorem 2.53 and the remark after it, some" to read
"Since V is open in a complete metric space, Theorem 2.53 and the remark after it show that some".

Page 121, 6 lines before Theorem 2.56. Change "$t \in [a,b]$" to "$t \in S$".

Page 122, displayed definition of $\delta_n(\epsilon)$. Change "$|f(x) - f(y)|$" to "$|f_n(x) - f_n(y)|$".

Page 122, paragraph "Thus we are to prove", line 3. Change "$\delta_{n_k}(\epsilon)$" to "$1/k$", and change "f_k" to "f_{n_k}" twice.

Page 122, lines -9 and -8. Change "f_{k_l}" to "$f_{n_{k_l}}$" six times.

Page 122, line -7. Change "f_k" to "f_{n_k}" twice.

Page 123, line 1. Insert the following sentence after "uniformly Cauchy."
"Redefining our indices, we may assume that $n_k = k$ for all k".

Page 126, line 11. Change "dense in $\mathcal{A}_{\mathbb{R}}$" to "dense in $C(S, \mathbb{C})$".

Page 126, Example 3, line 3. Change "S^{m-1}" to "S^{n-1}".

Page 128, line 6. Change "relation $\sim$ on X" to "relation $\sim$ on Cauchy(X)".

Page 130, Problem 2. Change "an open set that is at most countable" to "a countable open set". (This change is not logically necessary but may help avoid confusion, since "countable" and "at most countable" mean the same thing in this book.)

Page 132, Problem 21, line 5. Change "$\mathcal{A}^{\mathrm{cl}}$" to "$\mathcal{A}$".

Page 136, line -7. Change "$\inf\limits_{x \in \mathbb{F}^n}$" to "$\inf\limits_{M \geq 0}$".

Page 136, line -5. Change "$|T(cx)| = |c||x|$" to "$|T(cx)| = |c||T(x)|$".

Page 138, line -1. Change "$\Big(\sum\limits_{j=1}^{n} |x \cdot e_j|\Big)$" to "$\Big(\sum\limits_{j=1}^{n} |x \cdot e_j|^2\Big)$".

Page 141, line 15. Change "$(h_1, \ldots, h_m)$" to "$(h_1, \ldots, h_n)$".

Page 143, line 12. Change "$(t, b+h)$" to "$(t, b+k)$".

Page 148, display in first paragraph of proof of Proposition 3.12. Change "M^n" to "M^N" at the right end.

Page 156, line -3. For clarity, insert "$\varphi : \mathbb{R}^{n+m} \to \mathbb{R}^{n+m}$" after the word "function".

Page 157, after large displayed matrix. For clarity, insert the sentence
"The upper left block of $\varphi'(a,b)$ is the n-by-n identity matrix, and the lower right block is of size m-by-m."

Page 171, item (3), lines 5-6. Change "discontinuities ... as a function of x" to "discontinuities for each fixed x, and $(fI_U)_x(y)$ is therefore Riemann integrable as a function of y".

Page 171. line -3 of text. Change "$[a,b]$" to "$[A,B]$".

Page 176, line 6. Change "$\psi'(0)^{-1}$" to "$\psi_j'(0)^{-1}$".

Page 177, middle of page. Change the sentence "By the chain rule . . . nonsingular" to
"By the chain rule (Theorem 3.10), we have $\eta'(x) = \varphi'(x)(\psi^{-1})'(y)$, and this is nonsingular for x close enough to 0 ".

Page 234, statement of Proposition 5.1a. Change "$\bigcup_{n-1}^{N} E_n$" to "$\bigcup_{n=1}^{N} E_n$".

Page 235, line -10. Change "nonzero constant functions" to "constant functions".

Page 237, line 3. Change "$\mathbb{R}$" to "$\mathcal{R}$".

Page 237, line -17. Change "more than way" to "more than one way".

Page 238, line -7. Change "$[-\infty, -c]$" to "$[-\infty, c]$".

Page 238, line -5. Change "$[-c, +\infty]$" to "$[c, +\infty]$".

Page 239, line 5. Change "Problem 11 at the end" to "Problem 33 at the end".

Page 244, line 9. For clarity, change "absolutely convergent" to "absolutely convergent; this is a stronger condition since the sum has to be in $\mathbb{R}$".

Page 250, line 13. Change "pointwise f" to "pointwise limit f".

Page 263, line -4. Change "5.1f" to "5.1e".

Page 276, line 6. Change "shows that" to "shows for each standard basis vector u_j that".

Page 277, line -1. Change "$\sum_{j=1}^{n}$" to "$\sum_{j=1}^{m}$".

Page 278, line -8. Change "integrable" to "measurable on X with $|f|$ integrable on E".

Page 285, line 4 of Remarks. Change "an almost-everywhere convergent subsequence" to "a subsequence that converges pointwise almost everywhere".

Page 285, lines -8 and -7. Change so that these lines read:
"subsequence $\{f_{n_k}\}$ has the property that $\|f_{n_k} - f_{n_l}\| \le 2^{-\min\{k,l\}}$ for all $k \ge 1$ and $l \ge 1$. This proves the first conclusion of the theorem."

Page 286, line 2. Change "$\big(\int_X g_n \, d\mu\big)^{1/p}$" to "$\big(\int_X g_n^p \, d\mu\big)^{1/p}$".

Page 286, line 4. Change "$\big(\int_X g \, d\mu\big)^{1/p}$" to "$\big(\int_X g^p \, d\mu\big)^{1/p}$".

Page 286, line 7. Change "redefining the functions f_k" to "redefining the functions f_k as 0".

Page 286, line -8. Change "The redefined functions are" to "The sequence of redefined functions is".

Page 287, line 1. Change "Section 9" to "Section 8".

Page 289, line 3. Change "$= \|F\|_\infty$" to "$\le \|F\|_\infty$".

Page 290, line -3. Change "exists a Lebesgue" to "exists a bounded Lebesgue".

Page 300, line 25. Delete "of C".

Page 301, line -3. Change "Proposition 5.55d" to "Propositions 5.55c and 5.55d".

Page 302, line 1. Change "n-tuple" to "N-tuple".

Page 313, line -2. Change "$= 2\eta$" to "$\le 2\eta$".

Page 314, line -1. Change "F_{n_0}" to "$\varnothing$".

Page 317, line -17. Change "$|g(x) = h(x)|$" to "$|g(x) - h(x)|$".

Page 317, line -15. Change "is a vector subspace" to "contains a vector subspace". (This change is for clarity only; the closure actually is a vector subspace, but it is not necessary to prove this fact.)

Page 334, line 13. Change "with be" to "will be".

Page 339, line 1. Change "$F_N(x)$" to "$f_N(x)$".

Page 363, line 3. Change "$F'(k)$" to "$F_k'(x)$".

Page 365, line 6. Change "$f'(t)$" to "$f(t)$".

Page 366, second display on page. Change conditions at right side and one summation index on second line so that the display reads:

$$\text{“}S(x) = \begin{cases} \sum_{x_0 \le x_n \le x} c_n + \sum_{x_0 \le x_n < x} d_n & \text{if } x \ge x_0, \\ -\sum_{x < x_n \le x_0} c_n - \sum_{x \le x_n \le x_0} d_n & \text{if } x \le x_0, \text{”}. \end{cases}$$

Page 368, line 5 of proof of Theorem 7.9. Change "The function f is the derivative of F" to "The function f can be taken as the derivative of F".

Pages 368–369, Corollary 7.10 and Proposition 7.11. It is advisable to change the text's notation here, writing "μ_{cs}" in place of "μ_s". Then on page 420, μ_s will be the sum $\mu_s = \mu_{cs} + \mu_d$, and there will be no confusion about the meaning of μ_s in Section IX.4.

Page 370, line -2 of Section 2. Change
"By Corollary 7.4, $g = F' = f$. Hence $F(b) - F(a) = \int_a^b f(t)\,dt$" to
"By Corollary 7.4, $g = F' = f$ almost everywhere. Hence $F(b) - F(a) = \int_a^b g(t)\,dt = \int_a^b f(t)\,dt$".

Page 383, line 5 of proof of Theorem 8.9. Change "carried to L^2" to "carried to f".

Page 388, line -4. Change "and $\mathcal{F}(Af_j) \to \mathcal{F}(Af)$" to "and $\mathcal{F}(f_j) \to \mathcal{F}(f)$ and $\mathcal{F}(Af_j) \to \mathcal{F}(Af)$".

Page 389, line -1. Change "$\dfrac{1}{2L}\displaystyle\int_{-L}^{L}$" to "$\dfrac{1}{L}\displaystyle\int_{-L/2}^{L/2}$".

Page 392, first display on page. Include the term "$+\dfrac{\partial^2}{\partial t^2}$" on the right side.

Page 417, line -4. Change "$\tau_{t+h}f - \tau_t$" to "$\tau_{t+h}f - \tau_t f$".

Page 420, first paragraph of Section 4. If the notation has been changed in Corollary 7.10 and Proposition 7.11 in the way suggested above, then it can now be explained that $\mu_s = \mu_{cs} + \mu_d$.

Page 424, line −7. Change "σ-finite" to "σ-finite and $1 \le p \le \infty$" for clarity.

Page 452, line 20. Change "Thus c is in $S(x_0)$, and we" to "Since c is in $S(x_0)$, we".

Page 460, line −16. Change "covers X" to "covers X_s".

Page 466, line −5. Change "limit" to "limits".

Page 469, line −13. Change "$\mathcal{C} \times D$" to "$\mathcal{E} = \mathcal{C} \times D$" for clarity.

Page 469, line −12. Change "at the beginning of" to "earlier in".

Page 469, line −8. Change "domain value (A, β)" to "member (X, β) of $\mathcal{E} = \mathcal{C} \times D$".

Page 472, line 9. Change "$X/\sim$, and the property of" to "$X/\sim$. Property (ii) of".

Page 476, line −12. Change "distinct points of x" to "distinct points of X".

Page 478, line −17. Change "closure" to "closed set".

Page 478, line −5. Change "contain all its limit points" to "have no limit point in C".

Page 478, line −2. Change "Thus S has a limit point f not in S" to "Thus S has a limit point f in C".

Page 480, line 1. Change "$\sup_{y \in S}$" to "$\sup_{y \in X}$".

Page 481, Problem 7b, line 2. Change "$\sup_{m \in D,\ m \ge n}$" to "$\sup_{m \in D,\ m \ge n} t_m$".

Page 481, Problem 7b, line 3. Change "$\limsup_n t_n \ge t$" to "$\limsup_n t_n \le t$".

Page 488, line 23. For the words "elementary sets", use boldface instead of quotation marks.

Page 488, line −3. in the subscript, change "$0 \le f \le I_K$" to "$I_K \le f$".

Page 491, line 2 of Lemma 11.2. Change "of V" to "V of X", and change "$L \subseteq K \subseteq V$" to "$L \subseteq K$".

Page 493, line 19. Change "10.23" to "10.22".

Page 495, line before display of f and g at the bottom. Change "having with values" to "having values".

Page 498, line −11. Change "$L \cup L' \subseteq K \cup L'$" to "$L \cap L' \subseteq K \cap L'$".

Page 499, first display. Change "i_{k+1}" to "i_k" in two places.

Page 507, lines −7 and −6. Change "$\mathbb{R}^N$" to "X" in three places.

Page 508, lines 5 and 6. Change "$\mathbb{R}^N$" to "X" in two places.

Page 511, line 4. Change "$f(0)$" to "$\ell(0)$".

Page 514, line −5. Change "$-N$" to "-1".

Page 516, proof of Theorem 11.28. replace the third paragraph of the proof with the following:

"We shall define disjoint open sets U_i with $K_i \subseteq U_i$ for all i. We do so by constructing inductively on i for $1 \leq i \leq n$ disjoint open sets $U_1, \dots, U_i, V_i$ such that $K_j \subseteq U_j$ for $j \leq i$ and $K_{i+1} \cup \cdots \cup K_n \subseteq V_i$. Taking $i = n$ produces the required open sets $U_1, \dots, U_n$. For $i = 1$, Corollary 10.22 produces disjoint open sets U_1 and V_1 with $K_1 \subseteq U_1$ and $K_2 \cup \cdots \cup K_n \subseteq V_1$. Suppose that the construction has been carried out for stage i with $1 \leq i < n$. Using Corollary 10.22 for the locally compact Hausdorff space V_i, we choose disjoint open sets U_{i+1} and V_{i+1} of V_i with $K_{i+1} \subseteq U_{i+1}$ and $K_{i+2} \cup \cdots \cup K_n \subseteq V_{i+1}$. Then the disjoint open sets $U_1, \dots, U_{i+1}, V_{i+1}$ have $K_j \subseteq U_j$ for $j \leq i+1$ and $K_{i+2} \cup \cdots \cup K_n \subseteq V_{i+1}$. The inductive step is complete, and we have constructed disjoint open sets $U_1, \dots, U_n$ with $K_i \subseteq U_i$ for all i."

Page 541, line 11. Change "$X = L^1(S, \mu)$" to "the dual X^* of $X = L^1(S, \mu)$".

Page 556, lines 8–9. Change "whenever $y \in B$ and $z \in B$ are such that (x, y) and (y, z) are in f, then $y = z$" to
"there is exactly one $y \in B$ such that (x, y) is in f".

Page 557, lines 10–11. Change "range an unnamed set T, and $\bigcup_{x \in S} A_x$ is the set of all $y \in T$ such that y is in A_x for some $x \in S$" to
"range the set of all subsets of an unnamed set T, and $\bigcup_{x \in S} A_x$ is the set of all $y \in T$ such that y is in A_x for some $x \in S$".

Pages 574–576. The proof starting just below the middle of page 574 has a gap in its last paragraph, as was kindly pointed out by Qiu Ruyue. In addition, the order of the proof can be improved. The entire proof is to be replaced with the material listed under "A Long Correction" at the end of these pages.

Page 579, line -5. Change "$A \mapsto \{A\}$" to "$x \mapsto \{x\}$".

Page 581, number 1, line 1. Change "Let E" to
"The derivation for (a) is similar to the proof of Corollary 1.3. For (b), let E".

Page 581, number 1, line 8. Add at the end: "Doing so makes uses of (a)."

Page 583, number 13, lines 3 and 5. Change "$(0, \pi/2)$" to "$(-1, 1)$".

Page 589, number 21, line 1. Change "If x and y are given" to "If x and y are given with $x \neq y$".

Page 599, number 1 of Chapter V, line 1. Change "$0 \leq k \leq n$" to "$1 \leq k \leq n$".

Page 600, number 13. Replace with the following:

"13. Let $\mathcal{B}$ be the set of all subsets E of $X \times X$ such that there exists a set S_E in $\mathcal{A}$ with $E_x = S_E$ for all but countably many x in X. Every rectangle in $\mathcal{A} \times \mathcal{A}$ is in $\mathcal{B}$. In fact, there are two kinds of sets to check, sets $E = A \times B$ with A countable, in which case E_x is empty except for x in the countable set A, and sets $A^c \times B$ with A countable, in which case $E_x = B$ except for x in A. Also $\mathcal{B}$ is a σ-algebra. In fact, let sets E_n in $\mathcal{B}$ be given with associated sets S_{E_n}. Then $\big(\bigcup E_n\big)_x = \bigcup ((E_n)_x) = \bigcup S_{E_n}$ except when x is in the countable exceptional set for some n; also if E and S_E are given, then $(E^c)_x = (E_x)^c = (S_E)^c$ except when x lies in the exceptional set for E. Finally the diagonal D is not in $\mathcal{B}$ and therefore cannot be in $\mathcal{A} \times \mathcal{A}$. In fact, $D_x = \{x\}$ for each x, and there can be at most one x with $D_x = S_D$, whatever S_D is."

Page 607, line 5. Change "VI" to "$|VI|$".

A Long Correction

Pages 574–576. Replace the proof starting just below the middle of page 574 with the following:

Proof. A nonempty subset E of X wil be called *admissible* for purposes of this proof if $f(E) \subseteq E$ and if the least upper bound of each nonempty chain in E, which exists in X by assumption, actually lies in E. By assumption, X is an admissible subset of X. If x is in X, then the intersection of admissible subsets of X containing x is admissible. Let A_x be the intersection of all admissible subsets of X containing x. This is admissible, and since the set of all y in X with $x \le y$ is admissible and contains x, it follows that $x \le y$ for all $y \in A_x$. By hypothesis, X is nonempty. Fix an element a in X, and let $A = A_a$. The main step will be to prove that A is a chain.

To do so, consider the subset C of members x of A with the property that there is a nonempty chain C_x in A containing a and x such that

- $a \le y \le x$ for all y in C_x,
- $f(C_x - \{x\}) \subseteq C_x$, and
- the least upper bound of any nonempty subchain of C_x is in C_x.

The element a is in C because we can take $C_a = \{a\}$. If x is in C, so that C_x exists, let us use the bulleted properties to see that

$$A = A_x \cup C_x. \qquad (*)$$

We have $A \supseteq C_x$ by definition; also $A \cap A_x$ is an admissible set containing x and hence containing A, and thus $A \supseteq A_x$. Therefore $A \supseteq A_x \cup C_x$. For the reverse inclusion it is enough to prove that $A_x \cup C_x$ is an admissible subset of X containing a. The element a is in C_x, and thus a is in $A_x \cup C_x$. For the admissibility we have to show that $f(A_x \cup C_x) \subseteq A_x \cup C_x$ and that the least upper bound of any nonempty chain in $A_x \cup C_x$ lies in $A_x \cup C_x$. Since x lies in A_x, $A_x \cup C_x = A_x \cup (C_x - \{x\})$ and $f(A_x \cup C_x) = f(A_x) \cup f(C_x - \{x\}) \subseteq A_x \cup C_c$, the inclusion following from the admissibility of A and the second bulleted property of C_x.

To complete the proof of $(*)$, take a nonempty chain in $A_x \cup C_x$, and let u be its least upper bound in X; it is enough to show that u is in $A_x \cup C_x$. The element u is necessarily in A since A is admissible. Observe that

$$y \le x \quad \text{and} \quad x \le z \qquad \text{whenever } y \text{ is in } C_x \text{ and } z \text{ is in } A_x. \qquad (**)$$

If the chain has at least one member in A_x, then $(**)$ implies that $x \le u$, and hence the set of members of the chain that lie in A_x forms a nonempty chain in A_x with least upper bound u. Since A_x is admissible, u is in A_x. Otherwise the chain has all its members in C_x, and then u is in C_x by the third bulleted property of C_x.

This completes the proof of $(*)$. Let us now prove that if C_x and $C_{x'}$ exist with $x \le x'$ and $x \ne x'$, then

$$C_x \subseteq C_{x'}. \qquad (***)$$

In fact, application of $(*)$ to x' gives $A = A_{x'} \cup C_{x'}$. Intersecting both sides with C_x shows that $C_x = (C_x \cap A_{x'}) \cup (C_x \cap C_{x'})$. On the right side, the first member is empty by $(**)$, and thus $C_x = C_x \cap C_{x'}$. This proves $(***)$.

Let C be the set of all members x of A for which C_x exists. We have seen that a is in C. If we apply $(*)$ and $(**)$ first to a member x of C and then to a member x' of C, we see that either $x \leq x'$ or $x' \leq x$. That is, C is a chain.

Let us see that $f(C) \subseteq C$. If x is in C, then the set $D = C_x \cup \{f(x)\}$ certainly has a as a member. The second bulleted property of C_x shows that f carries $C_x - \{x\}$ into D, and also f carries x into D. Thus f carries $D - \{f(x)\}$ into D, and D satisfies the ssecond bulleted property of $C_{f(x)}$. If $\{x_\alpha\}$ is a chain in D with least upper bound u, there are two possibilities. Either u is $f(x)$, which is in D by construction, or u is in C, which contains the least upper bound of any nonempty chain in it. Thus u is in D, D satisfies the third bulleted property of $C_{f(x)}$, and $C_{f(x)}$ exists. In other words, $f(x)$ is in C, and $f(C) \subseteq C$.

Finally let us see that the least upper bound u of an arbitrary chain $\{x_\alpha\}$ in C, which exists in X by assumption, is a member of C. If $x_\alpha = u$ for some α, then $C_u = C_{x_\alpha}$ exists, and u is in C. So assume that $x_\alpha \neq u$ for all α. Our candidate for C_u will be $D = (\bigcup_\alpha C_{x_\alpha}) \cup \{u\}$. This certainly contains a. We check that D satisfies the second bulleted property of C_u. For each α, we can find a β with $x_\alpha \leq x_\beta$ and $x_\alpha \neq x_\beta$, since u is the least upper bound of all the x's. Then $(***)$ gives $C_{x_\alpha} \subseteq C_{x_\beta} - \{x_\beta\}$, and $f(C_{x_\alpha}) \subseteq f(C_{x_\beta} - \{x_\beta\}) \subseteq C_{x_\beta} \subseteq D$. Taking the union over α shows that D satisfies the second bulleted property of C_u.

To see that D satisfies the third bulleted property of C_u, let v be the least upper bound in A of a chain $\{y_\beta\}$ in C_u. If $v \neq u$, then v cannot be an upper bound of $\{x_\alpha\}$. So we can choose some x_{α_0} such that $v \leq x_{\alpha_0}$. Each y_β is $\leq v$, and thus each y_β is $\leq x_{\alpha_0}$. Referring to $(*)$, we see that all y_β's lie in $C_{x_{\alpha_0}}$. By the third bulleted property of $C_{x_{\alpha_0}}$, v is in $C_{x_{\alpha_0}}$. Thus v is in D, and D satisfies the third bulleted property of C_u. Consequently the least upper bound u of an arbitrary chain in C lies in C.

In short, C is an admissible set containing a, and it also is a chain. Since A is a minimal admissible set containing a, $C = A$ and also A is a chain. Let u be the least upper bound of A. We have seen that $f(A) \subseteq A$, and thus $f(u) \leq u$. On the other hand, $u \leq f(u)$ by the defining property of f. Therefore $f(u) = u$, and the proof is complete.

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