

Corrections to *Basic Algebra*

Page xii, line 3. Change “University” to “Universal”.

Page 3, after the statement of Proposition 1.2c. Insert “REMARK. Proposition 1.2c is sometimes called **Bezout’s identity**.”

Page 15, proof of Proposition 1.20. Replace sentences 3 through 5 with “Since $\lim_{x \rightarrow \pm\infty} P(x)/x^{2n+1} = 1$, there is some positive r_0 such that $P(-r_0) < 0$ and $P(r_0) > 0$.”

Page 34, item after (ii, a). Change “(a) $1v = v$ ” to “(b) $1v = v$ ”.

Page 40, proof of Theorem 2.8, line 3. Change “To see the middle inequality” to “To see the middle equality”.

Page 66, line above display in the middle of the page. Change “ $\det L = \begin{pmatrix} L \\ \Delta\Delta \end{pmatrix}$ ” to “ $\det L = \det \begin{pmatrix} L \\ \Delta\Delta \end{pmatrix}$ ”.

Page 76, line –8. Change “is an eigenvalue” to “is an eigenvector”.

Page 125, after proof of Proposition 4.4. Insert “REMARK. The proof of Proposition 4.4 exhibits a one-one correspondence between the subgroups of $\mathbb{Z}/m\mathbb{Z}$ and the positive integers k dividing m .”

Page 129, after the proof of Theorem 4.7. Insert “REMARK. Using the formula in Theorem 4.7 three times yields the conclusion that if H and K are subgroups of a finite group G with $K \subseteq H$, then $|G/K| = |G/H| |H/K|$.”

Page 134, line 6 of proof of Theorem 4.14. Change “ $(h_2^{-1}h_1^{-1}h_2)h_2^{-1}$ ” to “ $h_1^{-1}(h_1h_2^{-1}h_1^{-1})$ ”.

Page 136, Figure 4.2. Change the name of the map on the top arrow from “ φ_s ” to “ φ_{s_0} ”.

Page 150, line –16. Change “ $R = \mathbb{C}[X]$ ” to “ $R = \mathbb{C}, T = \mathbb{C}[X]$ ”.

Page 158, line 3 of Section 6. Change “Examples 5–9 of groups in Section 1 were” to “When $X = \{1, \dots, n\}$, the group $\mathcal{F}(X)$ is just the symmetric group $\mathfrak{S}_n$. Thus Examples 5–9 of groups in Section 1 are all”.

Page 162, line 11. Change “**isotropy subgroup** at p ” to “**isotropy subgroup** at p or **stabilizer** of G at p .”

Page 162, line 20. Insert a footnote after the clause “If $Y = Gp$ is an orbit”. The footnote reads, “Although the notation G_p for the isotropy subgroup and Gp for the orbit are quite distinct in print, it is easy to confuse the two in handwritten mathematics. Some readers may therefore prefer a different notation for one of them. The notation $Z_G(p)$ for the isotropy subgroup is one that is in common use; its use is consistent with the notation for the “centralizer” of an element in a group, which will be defined shortly. Another possibility, used by many mathematicians, is to write $G \cdot p$ for the orbit.”

Page 164, last line. Change “ $|G| = p^k$ ” to “ $|G_k| = p^k$ ”, and change “ $G_k \subseteq G_{k+1}$ ” to “ $G_k \subsetneq G_{k+1}$ ”.

Page 165, line 3. Change “4.39” to “4.38”.

Page 165, proof of Lemma 4.41. Change “ n ” to “ r ” in all 5 places it appears in the proof.

Page 167, footnote. Add a sentence at the end: “The normal subgroup goes on the open side of the $\ltimes$ and on the side of the subscript τ in $\times_\tau$.”

Page 171, last display. Change “ $\{1, (1\ 2)\}$ ” to “ $\{1, (1\ 2)(3\ 4)\}$ ”.

Page 183, remark at the bottom of the page. Add the sentence: “A consequence of (a) when $m \geq 1$ is that G has a subgroup of order p ; this special case is sometimes called **Cauchy’s Theorem in group theory**.”

Page 187, line 4 of “Proof of the remainder of Theorem 4.59.” Change “orbits of Γ under conjugation by G ” to “orbits in Γ under conjugation by G ”.

Page 200, Problem 18, line 2. Change “that is normal in M ” to “that is normal in N ”.

Page 240, Problem 12. Insert after the problem number: “(**Jordan–Chevalley decomposition**)”.

Page 241, Problem 13. Insert after the problem number: “(**Jordan–Chevalley decomposition**, continued)”.

Page 371, item (14), line 3. Change “whose multiplication is” to “whose ring operations are”.

Page 375, fourth new paragraph, line 1. Change “ M is a left R submodule” to “ M is a left R module”.

Page 379, third paragraph, line 1. Delete “nonzero”.

Page 380, statement of Proposition 8.6. Change “nonzero integral domain” to “integral domain”.

Page 387, line 7. Change “case $m = 0$ being trivial” to “case $m = 0$ following since r is not a unit”.

Page 390, line –14. Change “proof is application” to “proof is an application”.

Page 392, line –6. Insert at the end of the paragraph:

“We shall make computations with $c(A)$ as if it were a member of R , in order to keep the notation simple. To be completely rigorous, one should regard $c(A)$ as an orbit of the group $R^\times$ of units in R , using equality to refer to equality of orbits.”

Page 395, line 3. Replace this paragraph with the following shorter argument:

“In the second case, $P(X) = P$ has degree 0 and is prime in R . Put $R' = R(P)$ as in Proof #2 of Theorem 8.18. Then $A(X)B(X)$ maps to zero in the integral domain $R'[X]$, and hence $A(X)$ or $B(X)$ is in $P R[X]$.” $\square$

Page 395, proof of Corollary 8.22, line 2. Change “primitive” to “primitive; this adjustment makes use of the hypothesis that p does not divide a_N ”.

Page 397, Proposition 8.24. Insert a remark after the statement of the proposition:
 “REMARK. The proof will show that if M can be generated by n elements, then so can the unital R submodule.”

Page 399, Remark with Theorem 8.25. Add a sentence at the end:
 “Some people use the name “Fundamental Theorem of Finitely Generated Modules” to refer to Corollary 8.29 rather than to Theorem 8.25.”

Page 399, proof of Theorem 8.25, line 3. Change “is 0. Define” to
 “is 0. We argue as in the proof of Proposition 2.2. Define”.

Page 442, Problem 23. Change “an integer” to “a nonzero integer”.

Page 455, line 6. Change “ $X - X_n$ ” to “ $X - x_n$ ”.

Page 458, statement of Corollary 9.17. Insert a sentence at the end:
 “Conversely if $F(r) = F'(r) = 0$, then $(X - r)^2$ divides $F(X)$.”

Page 458, proof of Corollary 9.17. Remove the end-of-proof symbol, and insert a paragraph at the end:

“For the converse, let $F(r) = F'(r) = 0$. Proposition 4.28a shows that $F(X) = (X - r)G(X)$ and $F'(X) = (X - r)H(X)$. Differentiating the first equation gives $F'(X) = -rG(X) + G'(X) = (X - r)G(X) + (G'(X) - XG(X))$. Thus $H(X) = G'(X) - XG(X)$. The equality $F'(r) = 0$ shows that $H(r) = 0$. Thus $X - r$ divides $H(X)$.” $\square$

Page 468, footnote 2. Insert at the end:

“Computer calculations have shown that 2^{2^N+1} is not prime if $5 \leq N \leq 32$ ”.

Page 473, proof of Corollary 9.29. Insert an opening paragraph that says:

“The minimal polynomial of α_j over $\mathbb{k}(\alpha_1, \dots, \alpha_{j-1})$ divides the minimal polynomial of α_j over $\mathbb{k}$. If the second of these polynomials has distinct roots in a splitting field, so does the first. Thus (c) implies (b).”

Page 474, lines 6–7. Change “we obtain the equivalence of (a) and (c)” to
 “we see that (a) implies (c)”.

Page 475, proof of Theorem 9.34. Change the first word “We” to
 “We may assume that $\mathbb{k}$ is infinite because Corollary 4.27 shows that the multiplicative group of a finite field is cyclic. With $\mathbb{k}$ infinite, we”.

Page 475, proof of Theorem 9.34. Delete the last sentence of the proof.

Page 476, line 17. Change “some choice of c in $\mathbb{K}$ makes” to
 “we can choose c in $\mathbb{K}$ different from all the finitely many quotients $(\beta_i - \beta)(\alpha - \alpha_j)^{-1}$. For such a choice of c ,”.

Page 535, Problem 15. Change “ a is in $\mathbb{Q}$ and r is a member of $\mathbb{C}$ but not $\mathbb{Q}$ with $r^p = a$. Prove that” to

“ a is a member of $\mathbb{Q}$ such that $X^p - a$ has no root in $\mathbb{Q}$. If r is a member of $\mathbb{C}$ with $r^p = a$, prove that”

Page 605, answer to Problem 16. Replace “ n^2 ” by “ $n(n - 1)$ ” in two places.

Page 618, answer to Problem 18. Change “Section 7” to “Section 8”.

Page 670, answer to Problem 33, line 7. Change “ A ” to “ B ” twice.

Page 676, answer to Problem 15. Change “In (a) and (b), let” to “For (a) and (b), Lemma 9.45 shows that $X^p - a$ is irreducible over $\mathbb{Q}$. Hence $[\mathbb{Q}(r) : \mathbb{Q}] = p$. Let”.

Page 677, answer to Problem 17. Insert a sentence at the end:

“In other words, the only squares in $\mathbb{K}$ that lie in $\mathbb{k}$ are the obvious ones.”

Pages 703–717. The terms Bezout’s identity, stabilizer, Cauchy’s Theorem in group theory, and Jordan-Chevalley decomposition, which have all been introduced in this list of corrections, need to be added to the index.

12/21/2010