

**Corrections to  
Advanced Real Analysis  
and Some Remarks**

SHORT CORRECTIONS

Page 7, line 12. Change “ $X(t)T'(t)$ ” to “ $X(x)T'(t)$ ”.

Page 55, line 3. Change “ $e^{-2\pi i x \cdot y}$ ” to “ $e^{2\pi i x \cdot y}$ ”.

Page 124, line 7. Change “ $C(S_1)$ ” to “ $S_1$ ”.

Page 180, line –11. Change “ $\mathcal{S}'(U)$ ” to “ $\mathcal{S}'(\mathbb{R}^N)$ ”.

Page 181, lines 5–6. Move the clause “and it follows that  $\mathcal{S}(\mathbb{R}^N) \subseteq C^\infty(\mathbb{R}^N)$  has dense image” to the end of the previous sentence.

Page 209, Problem 10, line 2. Change “Suppose that  $T$  is a distribution” to “Suppose that  $T$  is a compactly supported distribution”.

REMARKS

Page 424, #6. This hint refers to the Stone–Weierstrass Theorem, which is to be applied to the one-point compactification of  $U$ . The version of the theorem to apply is not the one in Theorem 2.58 of *Basic*, but the one in Problems 21–23 on page 132 of *Basic*. The step of the hint that says “we check that the extension of the linear functional to  $C_{\text{com}}(U)$  is a positive linear functional” needs to be carried out. To do so, fix an open set with compact closure in  $U$ , and let  $\psi$  be a smooth function with values in  $[0, 1]$  that is compactly supported in  $U$  and is 1 on  $V$ . If  $\varphi$  is given in  $C_{\text{com}}(V)$ , choose a sequence  $\{f_n\}$  in  $C_{\text{textcom}}^\infty$  tending uniformly to  $\varphi$ . For any  $c > 0$ ,  $f_n + c\psi$  tends uniformly to  $\varphi + c\psi$  and is eventually  $\geq 0$ . Then it follows that the extended linear functional is  $\geq 0$  on  $\varphi + c\psi$ . Since  $c$  is arbitrary and then since  $V$  is arbitrary, the desired positivity follows.

Keng Wiboonton has pointed out that a simpler way of proceeding is to use Corollary 3.6a on page 61 instead of the Stone–Weierstrass Theorem to get at the approximation. The proof of this corollary is constructive, and positivity is preserved. Thus the supplementary argument is not needed.

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