INDEX

Abel, 521
abelian category, 238
abelian group
  divisible, 196
  torsion, 169
abelian Lie algebra, 78
absolute discriminant, 35, 267
absolute norm of ideal, 39, 273
absolute value, 289, 331
  archimedean, 289
  discrete, 338
  of idele, 390
  nontrivial, 332
  normalized, 383, 384, 385, 386
  trivial, 331
acyclic resolution, 219
additive category, 233
additive functor, 170, 178
adele, 389
adjoint, 252
affine algebraic set, 559
  dimension of, 566
  irreducible, 563
affine coordinate ring, 579
affine curve, irreducible, 529
affine Hilbert function, 621, 626
affine Hilbert polynomial, 625, 628
affine hypersurface, irreducible, 430, 562
affine local coordinates, 461
affine n-space, 455, 559
affine plane curve, 455
  irreducible, 430, 524, 562
affine plane line, 455
affine scheme, 642
affine variety, 429, 562
algebra, xxiii
  abelian Lie, 78
  central, 111
  central simple, 111
  crossed-product, 137
  cyclic, 122, 162, 163
  generalized quaternion, 121
  Lie, 77
  polynomial, 164
  semisimple, 80
  semisimple Lie, 78
  simple, 80
  simple Lie, 79
  solvable Lie, 78
  Weyl, 85
algebraic closure, separable, 434
algebraic set
  affine, 559
  irreducible affine, 563
  projective, 571
algebraically independent, 409
aligned primitive forms, 25
archimedean, 331, 333, 348
archimedean absolute value, 289
archimedean place, 383
archimedean valuation, 289
Artin product formula, 387, 390, 395
Artin reciprocity, 265
Artin's Theorem, 89
Artinian ring, 87
associated prime ideal, 446
associated translation, 622
associated vector subspace, 622
associative algebra
  semisimple, 80
  simple, 80
augmentation map, 149

Baer, 168
base field, 327
base space, 640
Bayer–Stillman ordering, 494
Bezout, 449

Bezout's Theorem, 453, 465, 471, 488
bidegree, 617
bifunctor, 223
bihomogeneous polynomial, 617
binary quadratic form, 3, 12
  similar, 74
birational, 595
birational map, 595
birationally equivalent, 595
Blichfeldt, 293
boundary, 172
boundary map, 172
boundary operator, 172
bounded sequence, 317
bracket, 78
Brauer equivalent, 124
Brauer group, 126
  relative, 127
Buchberger, 450
Buchberger's algorithm, 506

canonical class, 551
canonical divisor, 551
Cartan, E., 79
Cartan, H., 168
category
  abelian, 238
  additive, 233
  good, 169
Cauchy sequence, 317
Cayley, 77
central algebra, 111
central simple algebra, 111
centralizer, 114
chain complex, 171
  in abelian category, 240
  double, 257
  tensor product for, 258
chain map, 154, 155, 173
character
  Dirichlet, 62
  genus, 74
  multiplicative, 61
  principal Dirichlet, 62
Chase, 141
Chevalley, 165, 168
Chinese Remainder Theorem, xxiii, 30, 69, 106, 314, 341, 367, 480, 483
class field, Hilbert, 265
class field theory, 265
class group
  form, 28
  ideal, 42, 265, 299, 330
  idele, 393
class number, 299, 393
  Dirichlet, 7, 14
co-invariant, 209
co-invariants functor, 209
coboundary, 174
coboundary map, 174
coboundary operator, 174
cochain complex, 173
cochain map, 154, 174
cocycle, 174
codomain of morphism, 232
cohomology, 153, 174
  sheaf, 168, 171, 218, 643
coimage in abelian category, 239
cokernel, 175
  of morphism, 236
    universal mapping property, 236
common discriminant divisor, 272
common index divisor, 272, 287, 310, 371
commutator ideal, 78
complete presheaf, 641
complete valued field, 343
  equal-characteristic case, 398
  unequal-characteristic case, 398
completion, 342
  universal mapping property of, 343
complex, 171
  in abelian category, 240
  chain, 171
  cochain, 173
  double, 257
  flat, 259
complex place, 383
composition formula, 24
condition (C1), 165, 518
cone, 572, 633
conic, 458
conjugate, 266, 288, 383
connecting homomorphism, 185, 187
connecting morphism in abelian category, 248
convergent infinite product, 51
convergent sequence, 317

coordinate, 455, 559
  affine local, 461
coordinate hyperplane, 620
coordinate ring
  affine, 579
  homogeneous, 584
coordinate subspace, 619
coproduct, xxiii
correspondence, one-one, xxi
countable, xxi
Cramer, 448
Cramer's paradox, 449
Cramer's rule, 448
crossed-product algebra, 137
cubic, 458
  twisted, 562
cubic extension, pure, 280
cubic number field, 279
cubical singular chain, 172
cubical singular homology, 172
cup product, 256
curve
  affine plane, 455
  elliptic, 648
  irreducible, 604
  irreducible affine, 529
  irreducible affine plane, 430, 524, 562
  projective plane, 458
cycle, 172
cyclic algebra, 122, 162, 163
cyclotomic field, 309

decomposition group, 368
Dedekind, 77
Dedekind Discriminant Theorem, 275, 371, 379, 381
Dedekind domain, xxiv, 266
  extension of, xxiv, 327, 417
Dedekind example, 287, 310
Dedekind's Theorem on Differents, 376
defined at a point, 580, 585
degenerate, 172
degree, 153
  of divisor, 533
  of inseparability, 415
  residue class, 275, 354, 533
  total, 457
  transcendence, 413
derived functor, 204
  formation of, 205
  long exact sequence, 209, 214
Dickson, 122
different, 279
  relative, 279, 372
differential, 543, 547
differential form, 541
dimension
  of affine algebraic set, 566
  of affine variety, 563
  geometric, 565
  Krull, 424, 528, 529, 564
  of zero locus, 423
Diophantus, 1
direct product, restricted, 388
direct sum in additive category, 233
directed set, 438
Dirichlet, 2, 24, 77
Dirichlet box principle, 297
Dirichlet character modulo m, 62
Dirichlet class number, 7, 14
Dirichlet L function, 63
Dirichlet pigeonhole principle, 297
Dirichlet series, 56
Dirichlet Unit Theorem, 290, 292, 384, 390, 395
Dirichlet's Theorem, 7, 50
discrete, 290
discrete absolute value, 338
discrete valuation, 322
  defined over $\Bbbk$, 529
discriminant, 12
  absolute, 35, 267
  of commutative semisimple algebra, 382
  field, 35, 264, 267
  fundamental, 33
  of ordered basis, 267
  relative, 275, 381
discriminant divisor, 272
divisible abelian group, 196
divisible module, 251
division algorithm, generalized, 499
divisor, 532
  principal, 532
divisor class, 532, 549
domain of morphism, 232
dominant rational map, 595

Double Centralizer Theorem, 115
double chain complex, 257
dual of fractional ideal, 372

Eckmann, 168
Eilenberg, 168
Eisenstein, 12
Eisenstein polynomial, 402
elimination ideal, 512
Elimination Theorem, 512
elimination type ordering, 494, 512
elliptic curve, 648
enough injectives, 202
enough projectives, 202
epi, 233
epimorphism, 233
equal-characteristic case, 398
equivalence class of forms
  ordinary, 13
  proper, 13
equivalence of
  absolute values, 333
  completions, 383
  forms, 13, 32
    improper, 13
    proper, 13, 32
  ideals, 40, 298
    narrow, 40
    strict, 40, 298
  morphisms, 242
Euler, 1, 3, 9, 50
Euler product, 50, 54, 60
  first-degree, 60
Euler's Theorem, 516, 646
exact complex, 175
exact functor, 179
  left, 182
  right, 183
exact on injectives, 222
exact on projectives, 222
exact sequence, 175
  in abelian category, 240
  long, 187, 188
  short, 175
  split, 200
Exchange Lemma, 412
Ext functor, 223
extension normal 435
extension
  of Dedekind domain, xxiv, 327, 417
  of integrally closed domain, 610
  purely transcendental, 409
  of valued field, 358
Extension Theorem, 512

factor set, 133
  trivial, 135
Fermat, 1, 3, 9
field discriminant, 35, 264, 267
field of formal Laurent series, 347
field of fractions, xxiv
field polynomial, 266
fine sheaf, 218
finiteness of class number, 390
first-degree Euler product, 60
flabby sheaf, 218
flat complex, 259
flat module, 256
form
  binary quadratic, 3, 12
  class group, 28
  negative definite, 14
  positive definite, 14
  reduced primitive, 18, 21
forms, primitive aligned, 25
Fourier inversion formula for finite abelian groups, 61
fractional ideal, 321
  principal, 321
  relative dual, 372
free resolution, 152, 195
Freudenthal, 168
Frobenius element, 437
Frobenius's Theorem about division algebras over the reals, 118, 160
function field, 419, 528, 580, 582, 585, 587
  in one variable, 326, 382, 528, 529
  in r variables, 419
functor
  additive, 170, 178
  co-invariants, 209
  derived, 204
  exact, 179
  Ext, 223
  global-sections, 218
  homology-of-groups, 209

invariants, 208
left exact, 182
right exact, 183
Tor, 224
functorial, 177
functoriality
with long exact sequence, 191
of long exact sequence of derived functors, 215, 218
with snake diagram, 190
fundamental discriminant, 33
fundamental parallelotope, 293
Fundamental Theorem of Galois Theory, 443
fundamental unit, 36, 288

Galois, 77
Galois group, 434
gap sequence, 557
Gauss, 1, 3, 9, 24, 77
Gauss's group, 5, 28
Gelfand, 348
generalized division algorithm, 499
generalized quaternion algebra, 121
generalized resultant, 514
genus, 32, 539, 556, 557
principal, 33
genus character, 74
genus group, 33, 70, 73
geometric dimension, 565
germ, 584
global field, 382
global-sections functor, 218
good category, 169
graded lexicographic ordering, 493
graded monomial ordering, 627
graded reverse lexicographic ordering, 494
Grothendieck, 638
Gröbner, 450
Gröbner basis, 450, 497, 564
minimal, 508
reduced, 509

Haar measure, 385
Halphen, 450
Hamilton, 77
Hensel, 279
Hensel's Lemma, 349, 351, 353, 399
Herstein, 130
Hilbert, 404
Hilbert Basis Theorem, xxiv, 491, 560
Hilbert class field, 265
Hilbert function, 633
affine, 621, 626
Hilbert polynomial, 633
affine, 625, 628
Hilbert's Theorem 90, 71, 145
homogeneous coordinate ring, 584
homogeneous ideal, 458
homogeneous member of homogeneous coordinate ring, 585
homogeneous Nullstellensatz, 572
homogeneous polynomial, 457
homology, 153, 172
cubical singular, 172
simplicial, 172
homology-of-groups functor, 209
homomorphism, 78
connecting, 185, 187
inflation, 254
restriction, 254
of valued field, 342
homotopic, 154, 173, 174, 193, 198
homotopy, 173, 174, 193, 198
Hopf, H., 167
Hopkins, 92
Hurewicz, 167
hyperplane coordinate, 620
hypersurface, irreducible affine, 430, 562
hypersurface, irreducible projective, 573

ideal
fractional, 321
in Lie algebra, 078
principal fractional, 321
valuation, 322
ideal class group, 42, 265, 299, 330
idele, 390
idele class group, 393
idempotent, 91, 369
primitive, 369
image in abelian category, 239
Implicit Function Theorem, 428, 600
improper equivalence of forms, 13
independent, algebraically, 409
index, 272
ramification, 275, 354

inertia group, 370
inertia subfield, 368
inflation homomorphism, 254
inflation-restriction sequence, 254
injective, 195
  in abelian category, 241
injective module, 195
injective resolution, 199, 205
inseparable element, 414
integral closure, xxiv, 610
integral domain, xxiii
integral element, xxiv
integrally closed, xxiv
intersection multiplicity, 467, 474
intersection number, 467
invariant, 208
invariants functor, 208
inverse limit, 439
  standard, 439
inverse system, 438
irreducible affine algebraic set, 563
irreducible affine curve, 529
irreducible affine hypersurface, 430, 562
irreducible affine plane curve, 430, 524, 562
irreducible closed set, 564, 573
irreducible curve, 604
irreducible element, xxiii
irreducible ideal, 446
irreducible projective hypersurface, 573
irredundant, 446
isomorphic idempotents, 97
isomorphism, 78
  of valued field, 342
  of varieties, 591

Jacobi, 521
Jacobi identity, 78
Jacobi symbol, 68
Jacobson radical, 89

kernel of morphism, 235
  universal mapping property, 235
Koszul, 168
Kronecker, 77
Krull dimension, 424, 528, 529, 564
Kummer, 77
Kummer's criterion, 275
Künneth Theorem, 258

Lagrange, 1, 4
Langlands reciprocity, 265
largest domain, 583, 595
Lasker–Noether Decomposition Theorem, 446, 639
lattice, 290
Law of Quadratic Reciprocity, 3, 8
least common multiple, 501
left adjoint, 252
left Artinian ring, 87
left exact functor, 182
left Noetherian ring, 87
left semisimple ring, 81
Legendre, 1, 4
Legendre symbol, 8
Leibniz, 7
Leray, 168
Levi, E. E., 79
lexicographic ordering, 493
Lie algebra, 77
  abelian, 78
  semisimple, 78
  simple, 79
  solvable, 78
Lie subalgebra, 78
line
  affine plane, 455
  at infinity, 459
  projective, 458
Liouville, 521
local expression, 462
local morphism, 642
local ring, xxiv
local ring at a point, 580, 582, 585, 587
local/global approach, 371
localization, xxiv
locus of common zeros, 429, 559, 571
long exact sequence, 187, 188
  functoriality with, 191
  of derived functors, 209, 214
    functoriality with, 215, 218

Mac Lane, 168, 420
Macaulay, 627
maps of a good category, 169
matrix units, 101
member in abelian category, 242
minimal Gröbner basis, 508

Minkowski, 301, 302
Minkowski Lattice-Point Theorem, 293, 384
modules of a good category, 169
monic, 232
mono, 232
monomial, 457
  reduced, 646
monomial ideal, 619
monomial ordering, 493
  graded, 627
monomorphism, 232
morphism, 169
  of affine scheme, 642
  local, 642
  of ringed space, 642
  of varieties, 591
multiplicative, 60
  strictly, 60
multiplicative character, 61
multiplicity of a tangent line, 478

Nakayama's Lemma, xxiii, 120, 605, 606
narrow equivalence of ideals, 40
natural, 177
negative, xxi
negatively oriented, 40
neighbor, 21
neighbor on the left, 21
neighbor on the right, 21
nil left ideal, 89
nilpotent element, 89, 446
nilpotent left ideal, 80, 90
Noether Normalization Lemma, 612
Noether–Jacobson Theorem, 130
Noetherian, xxiv
Noetherian ring, 87
Noetherian topological space, 564
nonarchimedean, 331, 335, 338
nonarchimedean place, 383
nonsingular curve, 604
nonsingular point, 429, 600, 601
nontrivial absolute value, 332
norm, 165, 356
  of ideal, 39
    absolute, 273
normal extension, 435
normalized absolute value, 383, 384, 385, 386
Nullstellensatz, 404, 561
  homogeneous, 572
number field, xxiv
  cubic, 279
  cyclotomic, 309
  quadratic, 35, 69, 263, 269

Oka, 168
one-one correspondence, xxi
order, 532
order of vanishing, 474
ordering
  Bayer–Stillman type, 494
  graded lexicographic, 493
  graded monomial, 627
  graded reverse lexicographic, 494
  k-elimination type, 494, 512
  lexicographic, 493
  monomial, 493
  total, 493
  from tuple of weight vectors, 494
ordinary equivalence class of forms, 13
oriented, 40
orthogonal idempotents, 97, 369
Ostrowski, 348
Ostrowski's Theorem, 336

p-adic absolute value, 316
p-adic integer, 279, 318
$\mathfrak{p}$-adic integer, 346
p-adic metric, 316
p-adic number, 279, 316, 318
$\mathfrak{p}$-adic number, 346
parallelotope, 548
  fundamental, 293
Peirce decomposition, 95
perfect field, 418, 554
place, 383
plane, projective, 456
plane curve
  affine, 455
  irreducible affine, 430, 524, 562
  projective, 458
plane line, affine, 455
Plücker, 450
point, 455, 456, 459, 559
points at infinity, 459
pole part, 537
pole set, 581, 585

positive, xxi
positively oriented, 40
presheaf, 640
  complete, 641
primary ideal, 445
prime element, xxiii
prime ideal, xxiii
  associated, 446
primitive, 12
primitive form, reduced, 18, 21
primitive forms, aligned, 25
primitive idempotent, 369
primitively represent, 14
principal Dirichlet character, 62
principal divisor, 532
principal fractional ideal, 321
principal genus, 33
problem
  ideal-equality, 510
  ideal-membership, 507
  proper-ideal, 507
product, xxiii
profinite group, 441
projective, 192
projective algebraic set, 571
projective closure, 575
projective hypersurface, irreducible, 573
projective in abelian category, 241
projective limit, 439
projective line, 458
projective module, 192
projective n-space, 457, 570
projective plane, 456
projective plane curve, 458
projective resolution, 195, 205
projective transformation, 460
projective variety, 572
proper equivalence of
  forms over $\mathbb{Q}$, 32
  forms over $\mathbb{Z}$, 13
proper equivalence class of forms, 13
pullback, 242
pure cubic extension, 280
  type of, 281
purely inseparable element, 415
purely inseparable extension, 416
purely transcendental extension, 409
pushout, 202, 243

quadratic form
  binary, 3, 12
  similar, 74
quadratic number field, 35, 69, 263, 269
quadratic reciprocity, 3, 8, 68
quartic, 458
quasi-affine variety, 568
quasiprojective variety, 573
quaternion algebra, 121

radical
  of algebra, 80
  of ideal, 405
  Jacobson, 89
  of Lie algebra, 78
  Wedderburn–Artin, 89, 91
ramification index, 275, 354
ramified, 367
ramify, 264, 275, 308
rational function, 580, 585
rational map, 595
  dominant, 595
rational point, 455, 456, 457, 459
real place, 383
reciprocity
  Artin, 265
  Langlands, 265
  quadratic, 3, 8, 68
reduced Gröbner basis, 509
reduced monomial, 646
reduced norm, 165
reduced polynomial, 165
reduced primitive form, 18, 21
reduced trace, 165
reducible ideal, 446
regular at a point, 580, 582, 585
regular function at a point, 587
regular function on an open set, 580, 582, 587, 641
regular point, 429
relative Brauer group, 127
relative different, 279, 372
relative discriminant, 275, 381
relative dual of fractional ideal, 372
represent, 14
  primitively, 14
residue class degree, 275, 354, 533, 322
Residue Theorem, 543

resolution, 194
  acyclic, 219
  free, 152, 195, 195
  injective, 199, 205
  projective, 205
  standard, 149
restricted direct product, 388
restriction homomorphism, 254
resultant, 449, 451
  generalized, 514
Riemann, 521
Riemann hypothesis, 530
Riemann sphere, 328
Riemann surface, 522
Riemann zeta function, 52, 58
Riemann's inequality, 538
Riemann–Roch Theorem, 522, 551, 648
right adjoint, 252
right Artinian ring, 87
right exact functor, 183
right Noetherian ring, 87
right semisimple ring, 81
ring of formal power series, 347
ringed space, 642

S-polynomial, 502
scheme, 642
  affine, 642
  defined over a ring, 643
Schmidt, 422
Schreier, 168
Schur's Lemma, 83
section, 640
Segre embedding, 617, 646
Segre variety, 617
semisimple associative algebra, 80
semisimple Lie algebra, 78
semisimple module, xxii
semisimple ring, 81, 84
separable algebraic closure, 434
separable element, 414
separable extension, 415
separable polynomial, 414
separable semisimple algebra over a field, 109
separably generated extension, 419
separating transcendence basis, 419
sheaf, 168, 640
  cohomology, 168, 171, 218, 643
  fine, 218
  flabby, 218
  structure, 641
short exact sequence, 175
  in abelian category, 241
similar binary quadratic forms, 74
simple associative algebra, 80
simple Lie algebra, 79
simple module, xxii, 80
simple ring, 85
simplicial homology, 172
singular cube, 172
singular homology, 172
singular point, 429, 600, 601
Skolem–Noether Theorem, 113
snake diagram, 185, 261
  functoriality with, 190
Snake Lemma, 185, 248
solution of problem
  ideal-equality, 510
  ideal-membership, 507
  proper-ideal, 507
solvable Lie algebra, 78
spectral sequence, 171
spectrum, 639
split, 127
split exact sequence, 200
splitting field, 127
stalk, 640
standard inverse limit, 439
standard resolution, 149
standard subset, 622
Stickelberger's condition, 309
Stone, 638
strict equivalence of ideals, 40, 298
strictly multiplicative, 60
strong approximation property, 374
Strong Approximation Theorem, 372, 390, 391
structure sheaf, 641
summation by parts, 56

tangent lines, 478
tensor product of
  algebras, 104
  chain complexes, 258
  fields, 104
Theorem 90, Hilbert's, 71, 145
Tor functor, 224

Tornheim, 349
torsion abelian group, 169
torsion submodule, 257
total degree, 457
total ordering, 493
totally ramified, 367
trace, 165
transcendence basis, 409, 424
  existence, 411
  separating, 419
transcendence degree, 413
transcendence set, 409
translate of form, 26
triangular ring, 88
trivial absolute value, 331
trivial factor set, 135
twisted cubic, 562
type of pure cubic extension, 281

ultrametric inequality, 316, 331
unequal-characteristic case, 398
uniformizer, 323
uniformizing element, 323
unit, xxii, 36, 288
  fundamental, 36, 288
unital, xxii
Universal Coefficient Theorem, 261
universal mapping property of
  completion of valued field, 343
  cokernel, 236
  kernel, 235
unramified, 367

valuation, 322, 331
  archimedean, 289
  discrete, 322, 529
valuation ideal, 322
valuation ring, 322
valued field, 342
  complete, 343
  extension of, 358
  homomorphism of, 342
  isomorphism of, 342
variety, 590
  affine, 429, 562
  projective, 572
  quasi-affine, 568
  quasiprojective, 573
  as a scheme, 643
  Segre, 617

Weak Approximation Theorem, 340, 374
Wedderburn, 79, 86, 164
Wedderburn's Main Theorem, 94
Wedderburn's Theorem about finite division rings, 117, 160
Wedderburn's Theorem about semisimple rings, 83
Wedderburn–Artin radical, 89, 91
Weierstrass, 521
Weierstrass gap, 557
Weierstrass point, 557
Weierstrass valuation, 557
weight vectors, 494
Weil, 541, 543
Weyl algebra, 85

Zariski closure, 561, 578
Zariski topology, 560, 571
Zariski's Theorem, 431, 525, 601
zero locus, 455, 559, 571
zero member, 245
zero morphism, 233
zero object, 233
zeta function, 530
  Riemann, 52, 58