

Conference Schedule

Saturday, May 28

- 9:00am **John Hubbard** (Cornell University)
Minicourse 1: Parabolic blowups in complex dynamics and Kleinian groups
- 10:30 *Coffee break*
- 11:00am **Nikita Selinger** (University of Alabama at Birmingham)
Minicourse 2: Thurston's characterization theorem for branched covers
- 12:30 *break for lunch*
- 2:30pm **Núria Fagella** (Universitat de Barcelona)
Minicourse 3: Quasiconformal surgery and applications to transcendental dynamics
- 4:00 *Coffee break*
- 4:30pm **Xavier Gómez-Mont** (Centro de Investigación en Matemáticas)
Minicourse 4: Measuring the asymptotic distribution of holomorphic foliations
- 6:00 *Break for dinner*

Sunday, May 29

- 9:00am **John Hubbard** (Cornell University)
Minicourse 1: Parabolic blowups in complex dynamics and Kleinian groups
- 10:30 *Coffee break*
- 11:00am **Nikita Selinger** (University of Alabama at Birmingham)
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Minicourse 4: Measuring the asymptotic distribution of holomorphic foliations
- 6:00 *Break for dinner*

Monday, May 30

- 9:00am **Misha Lyubich** (Stony Brook University)
Pacman Renormalization and scaling of the Mandelbrot set at Siegel points.
- 10:00 *Coffee break*
- 10:30am **Carlos Cabrera** (Instituto de Matemáticas UNAM, Unidad Cuernavaca)
On cobordisms of rational functions
- 11:30 *break for lunch*
- 3:00pm **John Milnor** (Stony Brook University)
Curvature and résidu itératif
- 4:00 *Coffee break*
- 4:30pm **Saeed Zakeri** (Queens College/Graduate Center CUNY)
Rotation sets and polynomial dynamics
- 6:00 *cocktails*
- 7:00 *Welcome dinner*

Tuesday, May 31

- 9:00am **Xavier Buff** (Institut de Mathématiques de Toulouse)
Perturbations of maps tangent to $z \mapsto \bar{z}$
- 10:00 *Coffee break*
- 10:30am **Gamaliel Blé** (Universidad Juárez Autónoma de Tabasco)
Dynamics in a quartic polynomial family
- 11:30 *break for lunch*
- 2:30pm **Poster Session,**
- 3:00pm **Curtis McMullen** (Harvard University)
Surfaces in the space of surfaces
- 4:00 *Coffee break*
- 4:30pm **Jan Kiwi** (Pontificia Universidad Católica de Chile)
Cubic polynomials with one periodic critical point: irreducibility
- 5:30 *break for dinner*
- 8:00pm **Sarah Koch** (University of Michigan)
A disconnected deformation space of rational maps

Wednesday, June 1

- 9:00am *All-day excursion*

Thursday, June 2

- 9:00am **Mitsuhiro Shishikura** (Kyoto University)
Tropical Complex Dynamics
- 10:00 *Coffee break*
- 10:30am **Alberto Verjovsky** (Instituto de Matemáticas UNAM, Unidad Cuernavaca)
Modular groups over the quaternions and their corresponding four dimensional hyperbolic orbifolds and manifolds
- 11:30 *break for lunch*
- 2:30pm **Poster Session,**
- 3:00pm **Luna Lomanaco** (Universidade de São Paulo)
Satellite copies of the Mandelbrot set
- 4:00 *Coffee break*
- 4:30pm **Michael Yampolsky** (University of Toronto)
Renormalization and rotational attractors of two-dimensional dissipative dynamical systems
- 6:00 *cocktails*
- 7:00 *Farewell dinner*

Friday, June 3

- 9:00am **Pascale Roesch** (Aix Marseille Université)
Stretching rays for cubic polynomials
- 10:00 *Coffee break*
- 10:30am **Adam Epstein** (Warwick University)
Infinitesimal computations in arithmetic dynamics
- 11:30 *break for lunch*
- 3:00pm **Rodrigo Pérez** (Indiana University/Purdue University Indianapolis)
The secret combinatorial garden of Siegel
- 4:00 *Coffee break*
- 4:30pm **Eric Bedford** (Stony Brook University)
No smooth Julia sets for Hénon maps

Minicourses

John Hubbard: *Parabolic blowups in complex dynamics and Kleinian groups*
Saturday and Sunday, May 28 and 29. 9:00am

- Stability of Julia sets and Limit sets in the absence of parabolics
- Fatou coordinates and perturbed Fatou coordinates
- Geometric limits of Kleinian groups and the Chabauty topology
- The set-up for parabolic blow-ups: isomorphisms of perturbed Ecalle cylinders

Nikita Selinger: *Thurston's characterization theorem for branched covers*
Saturday and Sunday, May 28 and 29. 11:00am

In the early 1980s Thurston proved prominent theorems in the field of Complex Dynamics. His characterization theorem provides a topological criterion of whether a given combinatorics can be realized by a rational map. In our first lecture, we will explain the statement of the theorem and give examples of applications. In the second lecture, we will discuss the proof of the theorem and connections between Teichmüller theory and rational dynamics.

Nuria Fagella: *Quasiconformal surgery and applications to transcendental dynamics*
Saturday and Sunday, May 28 and 29. 2:30pm

In the first session of this minicourse we will explain the basics of quasiconformal surgery, a widely used technique in holomorphic dynamics which allows to construct holomorphic maps with prescribed dynamics. This survey will be accompanied by some examples and applications. In the second session we shall present some applications to prove results in transcendental dynamics. These may include, among other things, results concerning configurations of Herman rings for transcendental maps, and/or linearization of indifferent fixed points for the semistandard map.

Holomorphic Foliations is one of the ways in which Holomorphic Dynamics appears in nature. The simplest such objects to define are given by polynomial vector fields in 2 complex variables. The integral curves, which are tangent to the vector field, extend to the line at infinity and so define a holomorphic foliation in the projective plane $\mathbb{C}\mathbb{P}^2$ with a finite number of singularities. The leaves of the foliation are Riemann surfaces and the way they are attached give rise to the transversal (holomorphic) dynamics. For the generic such polynomial vector field, one has by Poincaré's linearization theorem a simple normal form at the singularities and a canonical hyperbolic metric on the leaves, which by Brunella's theorem has a subharmonic variation from leaf to leaf (Invent. Math., 152, 2003).

If you stand on a leaf, then you have a 2-dimensional horizon. Since evolution has provided us with a natural 1-dimensional time (either discrete or continuous), we are perplexed at having the possibility of moving with 2-dimensional time. One might suspect that going in different directions one will perceive different dynamic behaviour, visiting distinct parts of $\mathbb{C}\mathbb{P}^2$ with different statistics.

One way to resolve this perplexing situation, is to add 1 extra real dimension to the space, and consider the 5-dimensional real manifold $T^1\mathcal{F}$ obtained by the unit tangent vectors to the leaves, and move through the foliated geodesic flow of the canonical leafwise hyperbolic metric, returning to 1-dimensional time by advancing geodesically. This flow then is a hyperbolic flow restricted to the unit tangent bundle $T^1\mathcal{L}$ of each leaf, and so one has leafwise stable and unstable manifolds, and hence geodesics that share a common past or a common future.

One wants to understand how this hyperbolic behaviour interacts with the transversal dynamics. It is not exactly the setting of partial hyperbolic dynamics, since one is not making any hypothesis of the relative strength of the leafwise hyperbolic flow with the transverse dynamics. But anyway, one can use the integrability of the foliation (in the sense of Frobenius) and obtain a class of measures which captures the common far away past, that we call u -Gibbs states. Getting then rid of the extra dimension just introduced, one obtains a class of measures that are capturing the statistical behaviour of the leaves.

The objective of the short course is to enter into details of the above plan, and if curiosity beats you, then you may take a look at a recently posted paper I have with Christian Bonatti and Matilde Martínez in arXiv which does not restrict to $\mathbb{C}\mathbb{P}^2$, but avoids the singularity issue, or to a previous paper with Christian Bonatti and Ricardo Vila restricted to a simple class of polynomial vector fields, Riccati equations, that is related to another favorite subject: Finitely generated groups of Möbius transformations, found also in arXiv (and published in J. Dyn. Sys. and Ergodic Th., 30, 2010).

Abstracts of Lectures

listed alphabetically by speaker

Eric Bedford: *No smooth Julia sets for Hénon maps*

Friday, June 2. 4:30pm

We will discuss the polynomial automorphisms of \mathbb{C}^2 of positive entropy. These are the finite compositions of complex Hénon maps. We will show that the forward and backward Julia sets for these maps are never smooth. Thus this family of automorphisms contains no mappings which are analogous to the 1-dimensional case, where there are the special maps $z \mapsto z^d$ and the Tchebyshev maps.

Gamaliel Blé: *Dynamics in a quartic polynomial family*

Monday, May 20. 10:30am

We work with a quartic polynomial family obtained by the composition of two quadratic polynomials. This family has two complex parameters, and each polynomial has at most two different dynamics. We show some properties of connectedness locus and the Julia sets that appear in this family.

Xavier Buff: *Perturbations of maps tangent to $z \mapsto \bar{z}$*

Tuesday, May 31. 9:00am

Within the family $z \mapsto \bar{z}^2 + c$, Mukherjee, Nakane and Schleicher proved that the boundary of every hyperbolic component of odd period consists of three real analytic arcs connecting three cusp points. Using transversality techniques due to A. Epstein, we prove that the cusps are ordinary cusps.

This is joint work with J. Milnor and A. Bonifant

Carlos Cabrera: *On cobordisms of rational functions*

Monday, May 20. 10:30am

We discuss the question about geometric extensions of rational maps to the hyperbolic 3-space. Following these ideas we study the cobordism equivalence of rational maps. This is a joint work with Peter Makienko and Guillermo Sienna

Adam Epstein: *Infinitesimal computations in arithmetic dynamics*

Friday, June 2. 10:30am

Cohomology provides a framework for infinitesimal deformation theory in analytic and algebraic geometry. We consider endomorphisms of the projective line over an algebraically closed field. Congenial formalism available for arbitrary endomorphisms in characteristic 0 remains available for tamely ramified endomorphisms in positive characteristic. We pay particular attention to the relations among cycle multipliers, and to the rigidity of postcritically finite endomorphisms.

Jan Kiwi: *Cubic polynomials with one periodic critical point: irreducibility*

Tuesday, May 31. 4:30pm

Given $n \in \mathbb{N}$, in the moduli space of complex cubic polynomials with marked critical point those possessing a given critical point periodic of period n form a one dimensional algebraic set \mathcal{S}_n . We show how to apply results by M. Rees and G. Cui-L. Tan to prove that \mathcal{S}_n is irreducible, thus giving an affirmative answer to a question posed by Milnor. This is a joint work with Matthieu Arfeux.

Sarah Koch: *A disconnected deformation space of rational maps*

Tuesday, May 31. 8:00pm

Let $f : (\mathbb{P}^1, P) \rightarrow (\mathbb{P}^1, P)$ be a postcritically finite rational map with postcritical set P . In the 1980s, W. Thurston showed that the associated pullback map $\mathcal{T}_P \rightarrow \mathcal{T}_P$ has a unique fixed point in the Teichmüller space \mathcal{T}_P (provided that f is not a flexible Lattès map). In his PhD thesis, A. Epstein generalized Thurston's ideas and defined a deformation space associated to a rational map $f : (\mathbb{P}^1, A) \rightarrow (\mathbb{P}^1, B)$, where $A \subseteq B$, allowing for f which are not necessarily postcritically finite. By definition, the deformation space is the locus in \mathcal{T}_B where the pullback map $\mathcal{T}_B \rightarrow \mathcal{T}_A$ and the 'forgetful map' $\mathcal{T}_B \rightarrow \mathcal{T}_A$ agree. We exhibit a family of quadratic rational maps for which the associated deformation spaces all have infinitely many connected components. This is joint work with E. Hironaka.

Misha Lyubich: *Pacman Renormalization and scaling of the Mandelbrot set at Siegel points.*

Monday, May 20. 9:00am

In the 1980s Branner and Douady discovered a surgery relating various limbs of the Mandelbrot set. We put this surgery in the framework of "Pacman Renormalization Theory" that combines features of quadratic-like and Siegel renormalizations. Siegel renormalization periodic points (constructed by McMullen in the 1990s) can be promoted to pacman renormalization periodic points. We prove that these periodic points are hyperbolic with one-dimensional unstable manifold. As a consequence, we obtain the scaling laws for the centers of satellite components of the Mandelbrot set near the corresponding Siegel parameters.

It is a joint work with Dima Dudko and Nikita Selinger.

Curtis McMullen: *Surfaces in the Space of Surfaces*

Tuesday, May 31. 3:00pm

We will discuss the discovery of an unexpected, totally geodesic complex surface in moduli space.

John Milnor: *Curvature and résidu itératif*

Monday, May 20. 3:00pm

The résidu itératif, at an isolated parabolic fixed point, is a basic dynamic invariant. The talk will describe its relation to the curvatures of hyperbolic component boundaries at the associated points in parameter space.

Rodrigo Pérez: *The secret combinatorial garden of Siegel*

Friday, June 2. 3:00pm

We will review the original proof of existence of Siegel disks, and uncover the rich combinatorial structure in the linearization equation, which is at the heart of the argument.

Pascale Roesch: *Stretching rays for cubic polynomials*

Friday, June 2. 9:00am

We will discuss some work in progress with Shizuo Nakane concerning the landing of certain stretching rays in the space of cubic polynomials.

Mitsuhiro Shishikura: *Tropical Complex Dynamics*

Thursday, June 2. 9:00am

For a rational map with non-empty Fatou set, one can associate a piecewise linear map on a tree. From this “tree map”, on “tropicalized complex dynamics”, we can derive some information on whether certain type of dynamics can be realized, or at which degree such dynamics can be realized. This tree map is supposed to describe the degeneration of rational maps under the limit of quasiconformal deformation. In this talk, we will discuss various problems related to the tropical complex dynamics.

Alberto Verjovsky: *Modular groups over the quaternions and their corresponding four dimensional hyperbolic orbifolds and manifolds*

Thursday, June 2. 10:30am

Using the rings of Lipschitz and Hurwitz integers in the quaternion division algebra of the quaternions we define several Kleinian discrete subgroups of isometries of hyperbolic 4-space. These groups are a quaternionic versions of the classical modular group and Picard group. The quotients of hyperbolic 4-space by the action of these groups give examples of arithmetic orbifolds which are finitely covered by very interesting arithmetic hyperbolic 4-manifolds of finite volume

Michael Yampolsky: *Renormalization and rotational attractors of two-dimensional dissipative dynamical systems*

Thursday, June 2. 4:30pm

I will discuss our recent results and new directions in the application of renormalization techniques to the study of attractors of 2D maps. We have studied dissipative rotational attractors in two settings: Siegel disks of Henon maps and minimal attractors of diffeomorphisms of the annulus. Jointly with D. Gaidashev, we have extended renormalization of Siegel maps and critical circle maps to small 2D perturbations, and used renormalization tools to study the geometry of the attractors. In the Siegel case, jointly with D. Gaidashev and R. Radu we have proved that for sufficiently dissipative Henon maps with semi-Siegel points with golden-mean rotation angle, Siegel disks are bounded by topological circles. In the annulus case, jointly with D. Gaidashev, we have shown that for bounded type rotation numbers, critical annulus maps have a minimal attractor which is a C^0 , but not smooth, circle, answering a question of E. Pujals.

Saeed Zakeri: *Rotation sets and polynomial dynamics*

Monday, May 20. 4:30pm

Rotation sets under the angle-doubling map of the circle play a key role in Douady-Hubbard's study of the quadratic family and the Mandelbrot set. This talk will consider a higher-degree analog by investigating rotation sets under the multiplication by d map $t \mapsto dt \pmod{1}$ and their connection with degree d polynomial maps. We present a general theory of rotation sets that treats both rational and irrational cases in a unified fashion. In particular, using the notion of the "gap measure" we provide a simpler approach to the theorem of Goldberg-Tresser according to which minimal rotation sets are uniquely determined by their rotation number and deployment probability vector. As a main example, we explore the link between degree 3 rotation sets and one-dimensional families of cubic polynomials with a persistent indifferent fixed point.

Poster Presentations

listed alphabetically by author

M. C. Adrián Esparza Amador: *Families of Baker domains in functions with more than one essential singularity*

Domingo Gonzalez Martínez: *Topological Entropy*

Mikhail Hlushchanka: *Invariant graphs, tilings, and iterated monodromy groups*

Wolf Jung: *Quadratic matings*

Luis Manuel Martínez González: *Hausdorff dimension of Julia sets*

Rafaela de Sousa Martins: *About the Milnor fibrations for real analytic maps*

Josué Vázquez Rodríguez: *On the dynamics of the family $f_{c,b,z_0}(z) = c \sin(z) + b/(z - z_0)$ with b, c, z real numbers*