Stony Brook University

Simons Lectures in Mathematics – Spring 2020

Stanislav Smirnov

(University of Geneva, Skoltech)

2D percolation revisited

Percolation is a mathematical model for the filtering of a liquid through a porous material or the spread of a forest fire or an epidemic: the edges of some graph are declared open or closed depending on independent coin tosses, and then connected open clusters are studied. While simple to define, this model exhibits very complicated behavior, with non-trivial scaling exponents and dimensions. Centering on the 2D setting, we will discuss simple proofs of some important theorems, connection of percolation to other models, as well as remaining open questions.

In the first lecture, we will review the definition of the model, describe its properties and relation to other models, such as the Ising model of a ferromagnet. We will also give new short proofs of some important facts, such as the sharpness of phase transition — roughly speaking, if one increases proportion of open edges beyond some "critical value", the liquid suddenly starts percolating everywhere.

In the second lecture, we will discuss the phenomenon of conformal invariance, which occurs at criticality. It allows to connect percolation scaling limit to the Oded Schramm's SLE process and establish exact values of various dimensions and exponents, rigorously proving predictions from Conformal Field Theory. We will also present a short version of the conformal invariance proof, based on joint work with Mikhail Khristoforov,

In the third lecture, we will tell how a reaction-diffusion process in biological context leads to a rather surprising appearance of percolation and Ising-like colorings of the skin of Mediterranean lizards.

The three lectures can be attended independently.
Much of contents is accessible to advanced undergraduate students.

Lecture 1: Monday, March 2 – 4:00 pm, SCGP 103
Lecture 2: Tuesday, March 3 – 2:30 pm, SCGP 103
Lecture 3: Wednesday, March 4 – 2:30 pm, SCGP 103