# Conference Schedule

# Sunday, February 20

5:30pm Dinner (until 7:30 in Vistas Diningroom)

# Monday, February 21

- 7:00am Breakfast (until 9am in Vistas Dining Room)
- 9:00am **John Hubbard** (Cornell University & Université Aix-Marseille) Limits of quadratic polynomials as dynamical systems and parabolic blow-ups (joint work with Ismael Bachy, much inspired by Adam Epstein)
- 10:00am Break (refreshments in 2<sup>nd</sup> floor Galleria)
- 10:30am **Stanislav Smirnov** (Université de Genève & St. Petersburg State University) *Quasiconformal maps and harmonic measure*
- 11:30am Lunch (until 1:30pm in Vistas Dining room)
  - 1:30pm **Pascale Roesch** (Université Paul Sabatier Toulouse III) The parabolic Mandelbrot set
- 2:30pm Sebastian van Strien (University of Warwick) Monotony of entropy
- **3:30pm** *Break* (refreshments in 2<sup>nd</sup> floor Galleria)
- 4:00pm William Thurston (Cornell University) Real Polynomial Entropy
- 5:30pm Dinner (until 7:30 in Vistas Diningroom)
- 7:30pm **Nikita Selinger** (Jacobs University Bremen) Boundary behavior of Thurston's pullback map and Pilgrim's conjecture
- 8:00pm **Jeremy Kahn** (Stony Brook University) The Surface Subgroup Theorem and the Ehrenpreis conjecture

## **Tuesday, February 22**

- 7:00am Breakfast (until 9am in Vistas Dining Room)
- 9:00am **Curtis McMullen** (Harvard University) Moduli spaces for holomorphic dynamics on the unit disk
- 10:00am Break (refreshments in 2<sup>nd</sup> floor Galleria)
- 10:30am **Serge Cantat** (Université de Rennes) Complex projective varieties with a large group of holomorphic diffeomorphisms
- 11:30am Lunch (until 1:30pm in Vistas Dining room)
- 1:30pm **Sarah Koch** (Harvard University) Moduli space maps and compactifications: a worked out example of mating (Joint work with Xavier Buff and Adam Epstein)
- 2:30pm **Nessim Sibony** (Université Paris-Sud-Bat 425) Dynamics of holomorphic foliations by Riemann surfaces
- **3:30pm** *Break* (refreshments in 2<sup>nd</sup> floor Galleria)
- 4:00pm **Alberto Verjovsky** (Universidad Nacional Autónoma de México) Holomorphic dynamical systems whose orbit spaces give new examples of compact complex manifolds
- 6:00pm Banquet (cash bar) in Kinnear Center 103/Husky Great Hall
- 7:00pm Banquet (meal) in Kinnear Center 103/Husky Great Hall

## Wednesday, February 23

7:00am Breakfast (until 9am in Vistas Dining Room)

Informal discussions throughout the day.

- 5:30pm Dinner (until 7:30 in Vistas Diningroom)
- 7:00pm **Mitsuhiro Shishikura** (Kyoto University) Renormalization for irrationally indifferent fixed points of holomorphic functions
- 8:00pm **Carsten Lunde Petersen** (Roskilde University) Conformally natural extensions revisited
- 8:30pm **Anna Zdunik** (University of Warsaw) Equilibrium measures in holomorphic dynamics– stochastic properties

### Thursday, February 24

- 7:00am Breakfast (until 9am in Vistas Dining Room)
- 9:00am Arnaud Chéritat (CNRS/Institut de Mathmatiques de Toulouse) About Zhang's premodels for Siegel disks of quadratic rational maps
- 10:00am Break (refreshments in 2<sup>nd</sup> floor Galleria)
- 10:30am **Jan Kiwi** (Pontificia Universidad Católica de Chile) Rescaling limits of complex rational maps
- 11:30am Lunch (until 1:30pm in Vistas Dining room)
  - 1:30pm **Charles Favre** (École Polytechnique) Non archimedean Montel Theorem
- 2:30pm Rotislav Grigorchuk (Texas A&M University) On Milnor's Problem on growth of groups
- **3:30pm** *Break* (refreshments in 2<sup>nd</sup> floor Galleria)
- 4:00pm **William Goldman** (University of Maryland) *Two papers which changed my life: Milnor's seminal work on flat manifolds*
- 5:30pm Dinner (until 7:30 in Vistas Diningroom)
- 7:30pm **Daniel Meyer** (University of Helsinki) Invariant Peano curves of expanding Thurston maps
- 8:00pm Vladlen Timoin (Higher School of Economics, Faculty of Mathematics) Partial semi-conjugacies between rational functions
- 8:30pm **Hiroyuki Inou** (Kyoto Univeristy) Bifurcation locus of the cubic polynomial family

#### Friday, February 25

- 7:00am Breakfast (until 9am in Vistas Dining Room)
- 9:00am Adam Epstein (University of Warwick) Transversality principles in holomorphic dynamics
- 10:00am Break (refreshments in 2<sup>nd</sup> floor Galleria)
- 10:30am **John Smillie** (Cornell University) Billiards in polygons: an example
- 11:30am Lunch (until 1:30pm in Vistas Dining room)

Informal discussions from 12:30-3:30.

#### Abstracts of Lectures

listed alphabetically by speaker

**Serge Cantat**: Complex projective varieties with a large group of holomorphic diffeomorphisms

Tuesday, February 22. 9:00am

Let M be a complex projective variety and let Aut(M) be the group of holomorphic diffeomorphisms of M. I shall discuss the "size" of the group Aut(M) and describe recent results that characterize projective varieties M with largest possible Aut(M).

Arnaud Chéritat: About Zhang's premodels for Siegel disks of quadratic rational maps Thursday, February 24. 9:00am

Adam Epstein: Transversality principles in holomorphic dynamics Friday, February 25. 9:00am

The moduli space of all degree *D* rational maps is an orbifold of dimension 2D - 2. We present a language for describing dynamically natural subspaces, for example, the loci of maps having

- specified critical orbit relations,
- cycles of specified period and multiplier,
- parabolic cycles of specified degeneracy and index,
- Herman ring cycles of specified rotation number,

or some combination thereof. We present a methodology for proving the smoothness and transversality of such loci. The natural setting for the discussion is a family of deformation spaces arising functorially from first principles in Teichmüller theory. Transversality flows from an infinitesimal rigidity principle (following Thurston), in the corresponding variational theory viewed cohomologically (following Kodaira-Spencer). Results for deformation spaces may then be transferred to moduli space. Moreover, the deformation space formalism and associated transversality principles apply more generally to finite type transcendental maps.

Charles Favre: Non archimedian Montel Theorem

Thursday, February 24. 1:30pm

We shall present a natural analog of Montel's theorem in a non-archimedean context. This is joint work with J. Kiwi and E. Trucco.

**William Goldman**: Two papers which changed my life: Milnor's seminal work on flat manifolds

Thursday, February 24. 4:00pm

This talk will survey recent developings following Milnor's contributions to manifolds with flat structures. Milnor's 1958 paper, "On the existence of a connection with curvature zero," began a development of the theory of characteristic classes of flat bundles, foliations, and bounded cohomology. His 1977 paper, "On fundamental groups of complete affinely flat manifolds," clarified the theory of complete affine manifolds, and set the stage for startling examples of Margulis of 3-manifold quotients of Euclidean 3-space by free groups of affine transformations. These structures intimately relate to noncompact hyperbolic surfaces. This part of the lecture, representing joint work with Charette, Drumm, Fried, Labourie and Margulis, will describe the topology, dynamics, and classification of these 3-manifolds.

Rotislav Grigorchuk: On Milnor's Problem on growth of groups		
Thursday, F	ebruary 24.	2:30pm

We will give an overview of results around Milnor's problem on growth of finitely generated groups. We will start with the polynomial growth case and then will switch to the case of intermediate growth (between polynomial and exponential). Numerous results about groups of intermediate growth will be formulated and concrete examples of such groups will be provided. Building on these examples, we will try to give a short introduction to the theories of Self-similar groups, Branch groups, and Nekrashevych Iterated Monodromy Groups. A number of open problems will conclude the talk.

**John Hubbard**: Limits of quadratic polynomials as dynamical systems and parabolic blow-ups(joint work with Ismael Bachy, much inspired by Adam Epstein)

Monday, February 21. 9:00am

Write  $p_c(z) = z^2 + c$ , and let  $K_c$  be the filled in Julia set. Give the set C of compact subsets of  $\mathbb{C}$  the Hausdorff topology. It is well known that the map  $c \mapsto K_c$  is not continuous at those c where  $p_c$  has a parabolic cycle.

What is the closure of the set of  $K_c$  in the Hausdorff metric? I will try to answer this question using a construction I call the *parabolic blow-up* and the projective limit of all finite parabolic blow-ups. In particular, what is the cohomology of the space? What is the proper transform of the boundary of the cardioid?

**Hiroyuki Inou**: *Bifurcation locus of the cubic polynomial family* Thursday, February 24. 8:30pm

In the past few years, Ushiki has been producing computer pictures of Julia sets in  $\mathbb{C}^2$ . Hence it is natural to think about visualization of two-dimensional parameter spaces for one-dimensional complex dynamics. I would like to present some attempts to see how the bifurcation locus of cubic polynomials looks like.

Jeremy Kahn: The Surface Subgroup Theorem and the Ehrenpreis conjecture Monday, February 21. 8:00pm

We prove that there is a hyperbolic surface S such that for any closed hyperbolic 2 or 3-manifold M, and  $\epsilon > 0$ , there is a finite cover  $\hat{S}$  of S, and a map  $f : \hat{S} \to M$  that is locally within  $\epsilon$  of being an isometric immersion. When dim M = 3 this implies that  $\pi_1(M)$  has a surface subgroup, and when dim M = 2 this is the Ehrenpreis conjecture.

In either case, the surface f(S) is constructed by putting together immersed pairs of pants in M, and in both cases we can construct a collection of good pants that are evenly distributed around every closed geodesic that appears as a boundary. If dim M = 3 then we can immediately assemble these pants, with a twist, to form the desired surface f(S).

In the case where dim M = 2, there may be more pants on one side of a geodesic than the other. In order to determine how to correct the collection of pants, we develop the "good pants homology" of good curves modulo the boundaries of good pants, and show through a series of algebraic identities that it is equivalent to the standard homology.

This is joint work with Vladimir Markovic.

Jan Kiwi:	Rescaling	limits	of compl	lex rational	maps
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Thursday, February 24. 10:30am

The emphasis of the talk will be on rescaling limits of complex rational maps (in one variable) and its Puiseux series dynamics counterpart: non-classical repelling periodic points. Rescaling limits appear in the literature as a relevant tool to study parameter spaces of rational maps near "infinity". We will illustrate these applications mainly in the context of quadratic rational maps. Then we will focus on our main result. Namely, given a sequence of rational maps diverging to infinity in parameter space, we give an upper bound for the number of "dynamically distinct" rescaling limits which are not post-critically finite. The main idea of the proof is to establish a correspondence between rescaling limits and some repelling periodic orbits in the Berkovich projective line over the field of Laurent series.

**Sarah Koch**: Moduli space maps and compactifications: a worked out example of mating (Joint work with Xavier Buff and Adam Epstein)

Tuesday, February 22. 1:30pm

We begin with a rational map  $F : \mathbb{P}^1 \to \mathbb{P}^1$  which is a mating of two critically finite hyperbolic polynomials of degree *d*. We analyze the associated map on the moduli space  $g : \mathcal{M}_P \dashrightarrow \mathcal{M}_P$ , where *P* is the postcritical set of *F*; we ultimately construct a "natural" compactification of  $\mathcal{M}_P$  for which the map *g* is algebraically stable.

**Curtis McMullen**: *Moduli spaces for holomorphic dynamics on the unit disk* Tuesday, February 22. 9:00am

Daniel Meyer: Invariant Peano curves of expanding Thurston maps Thursday, February 24. 7:30pm

A Thurston map is a postcritically finite branched covering map  $f: S^2 \to S^2$ . We consider such maps that are expanding in a suitable sense. We show that a suitable iterate  $F = f^n$  is semiconjugate to  $z^d: S^1 \to S^1$ . This means that there is a Peano curve  $g: S^1 \to S^2$  (onto) such that  $F(g(z)) = g(z^d)$ , where  $d = \deg F$ . This generalizes a result by Milnor and corresponds to a result by Cannon-Thurston in the group case.

Carsten Lunde Petersen: Conformally natural extensions revisited Wednesday, February 23. 8:00pm

In this talk I revisit the notion of conformal barycenter of a measure on  $\mathbb{S}^n$  as defined by Douady and Earle. The aim is to extend rational maps from the Riemann sphere to the (hyperbolic) three ball and thus to the three sphere by reflection. The construction which was pioneered by Douady and Earle in the case of homeomorphisms actually gives extensions not only for rational maps, but also for more general maps such as entire transcendental maps. And finally it works in any dimension.

Pascale Roesch: The parabolic Mandelbrot set

Monday, February 21. 1:30pm

In the moduli space of rational maps of degree 2, sit the slices  $Per_1(\lambda)$  of classes of rational maps having a fixed point of multiplier  $\lambda$ . The slice  $Per_1(0)$  corresponds to the set of quadratic polynomials. The connectedness locus  $M_{\lambda}$  in  $Per_1(\lambda)$  is, when  $|\lambda| < 1$ , a deformation of  $M_0$ , the Mandelbrot set. In a joint work with C. L. Petersen we study the connectedness locus  $M_1$  in  $Per_1(1)$  and prove that there exists a homeomorphism between  $M_0$  and  $M_1$  that preserves the dynamics.

Nikita Selinger: Boundary behavior of Thurston's pullback map and Pilgrim's conjecture Tuesday, February 22. 7:30pm

We define explicitly the extension to the augmented Teichmüller space. We characterize then the dynamics of Thurston's pullback map near invariant strata on the boundary of the augmented Teichmüller space. We use the obtained classification to simplify the proofs of Thurston's theorem and Canonical Obstruction theorem due to Pilgrim as well as to prove some further results about the behavior of the Thurston pullback map on the boundary, in particular, Pilgrim's conjecture.

**Mitsuhiro Shishikura**: Renormalization for irrationally indifferent fixed points of holomorphic functions

#### Wednesday, February 23. 8:00pm

An irrationally indifferent fixed point of a holomorphic function is a source of interesting and delicate phenomena in complex dynamics. When the rotation number is of sufficiently high type (i.e. the coefficients in continued fraction are large), the first return map to a certain fundamental region defines another irrationally indifferent function, which is called the near-parabolic renormalization. In a joint work with H. Inou, we found an invariant space of functions including quadratic polynomials and the renormalization acts as a hyperbolic map with respect to the metric induced from a Teichmüller space. We will discuss some applications of this renormalization.

Nessim Sibony: Dynamics of holomorphic foliations by Riemann surfaces Tuesday, February 22. 2:30pm

The goal is to describe statistical behavior of leaves of a possibly singular foliation by Riemann Surfaces. The main example to have in mind is the case of a polynomial vector field in the complex Euclidean space of arbitrary dimension. It induces a (singular) foliation by Riemann surfaces in the projective space. Generically (on the foliation) the leaves are covered by the unit disc and there is no measure invariant by holonomy. There are however harmonic measures (positive ddc closed currents) directed by the foliation.

I will discuss the following topics.

- 1. Geometric ergodic theorem, à la Birkhoff. Averages of leaves converge towards the ddc closed current *T*, for *T* almost every leaf.
- 2. Heat equation with respect to a harmonic measure. Since the leaves do not have bounded geometry because of singularities, we have to develop a different approach for the heat diffusion.
- 3. Transverse regularity for the Poincaré metric.
- 4. Entropy for hyperbolic Riemann Surface lamination.

This is joint work with T.C. Dinh and V.A. Nguyen.

John Smillie: Billiards in polygons: an example

Friday, February 25. 10:30am

There has been a great deal of activity in the field of Teichmuller dynamics. We will survey some of the techniques and results of this field focussing on the concrete example of the regular octagon.

Stanislav Smirnov: Quasiconformal maps and harmonic measure Monday, February 21. 10:30am

In 1994, Kari Astala established sharp bounds for the distortion of area and Hausdorff dimension by quasiconformal maps. His elegant proof used ideas from complex dynamics and provided examples of extremal sets. E.g., for a set *E* of dimension 1 and a *k*-quasiconformal map  $\phi$ ,

$$\operatorname{HDim}(\phi E) \le 1 + k ,$$

and the bound is sharp. A sometimes better estimate was proved by Becker and Pommerenke for quasicircles (when E is a circle), which led Astala to conjecture that in this case

$$\operatorname{HDim}(\phi E) \le 1 + k^2$$

We established this inequality using Astala's original proof along with a skew-symmetric decomposition of quasiconformal maps, but the question of its sharpness remains open.

Moreover, it turns out to be intimately related to multifractal properties of the harmonic measure, as well as dimensional estimates for quasi-Fuchsian groups. We will give an exposition of related problems, with an emphasis on open questions.

Partially based on joint work with Kari Astala and Istvan Prause.

Sebastian van Strien: Monotony of entropy

Monday, February 21. 2:30pm

An old conjecture is whether the topological entropy of the interval map  $x \mapsto ax(1-x)$  is an increasing function of a. This was solved in the 80's by (amongst others) Milnor and Thurston. In the early 90's Jack Milnor generalized the original conjecture to general polynomial maps of the interval, asserting that each locus of constant topological entropy in parameter space a connected set. For cubic maps this conjecture was solved by Milnor & Tresser in 2000. A few years ago, Henk Bruin and I proved the general result. This talk will describe some of the main ideas and possible extensions.

Vladlen Timorin: *Partial semi-conjugacies between rational functions* Thursday, February 24. 8:00pm

We consider rational functions as dynamical systems on the Riemann sphere; and discuss partially defined semi-conjugacies between non-hyperbolic and hyperbolic rational functions.

#### William Thurston: Real Polynomial Entropy

Monday, February 21. 4:00pm

This topic is an outgrowth of work done with Jack Milnor many years ago. I will discuss two aspects:

- (a) A real number a > 1 is the topological entropy of a critically finite map of an interval to itself if and only if it is the log of a weakly Perron number, i.e. it is the root of an integer monic polynomial with no other roots of larger modulus.
- (b) There is a Mandelbrot-like set such that all Galois conjugates of exp(entropy) for a critically finite quadratic map are in the set. The picture below is from a sample size of about 1,000,000,000. There are a number of other related pictures.



This is a composite of the kneading root pictures (black), the limits of roots of integer polynomials with coefficients  $\pm 1$  (black + green), and the limits of roots of integer polynomials with coefficients 1, 0, -1, which is the connectedness locus for a certain class of affine fractals (black + green + yellow).

**Alberto Verjovsky**: Holomorphic dynamical systems whose orbit spaces give new examples of compact complex manifolds

Tuesday, February 22. 4:00pm

We will give describe a panorama of certain compact, connected, complex manifolds, which are not symplectic (therefore not Kähler) and which are the orbit spaces of certain systems of linear differential equation over the complex numbers. These manifolds can have very complicated topology but under a condition of integrality they are Seifert fibrations over toric manifolds.

These results were obtained in collaboration with Laurent Meersseman (Université de Bourgougne, France).

Anna Zdunik: Equilibrium measures in holomorphic dynamics– stochastic properties Wednesday, February 23. 8:30pm

In this talk, I will outline the construction of natural invariant measures (equilibrium measures for Hölder continuous potentials) for iterations of holomorphic maps of complex projective spaces. The stochastic properties of these measures will be discussed. The method of proof gives also several interesting corollaries for (the best understood) one-dimensional case.

This is a joint work with Mariusz Urbanski and Michal Szostakiewicz.